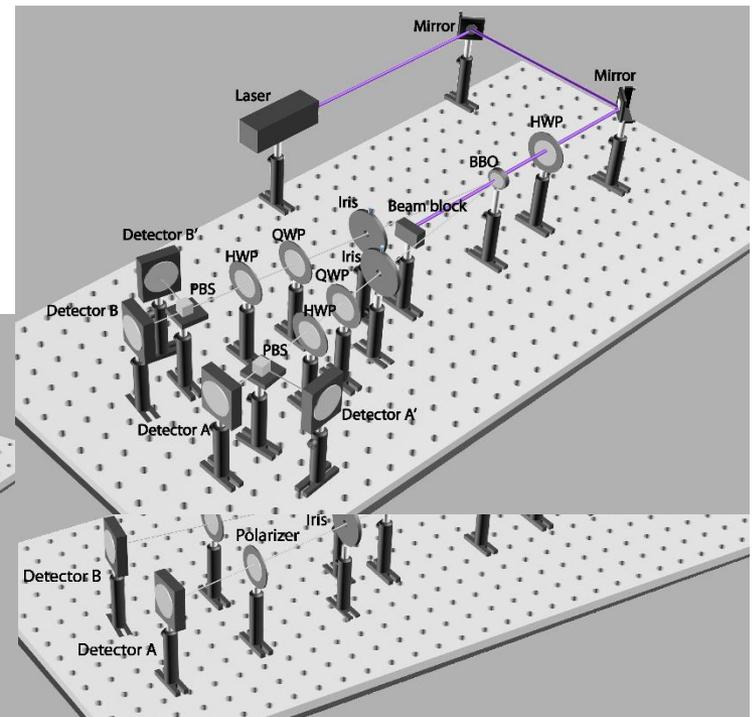
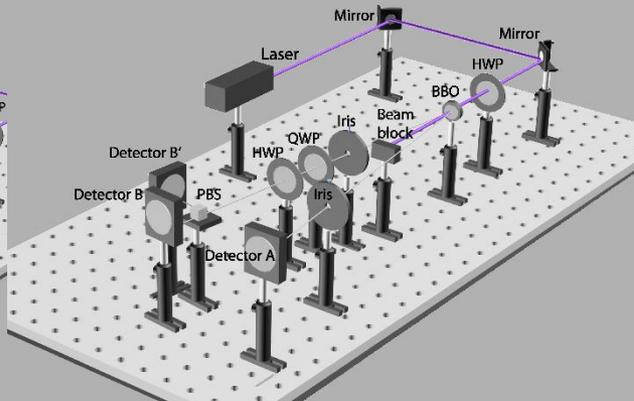
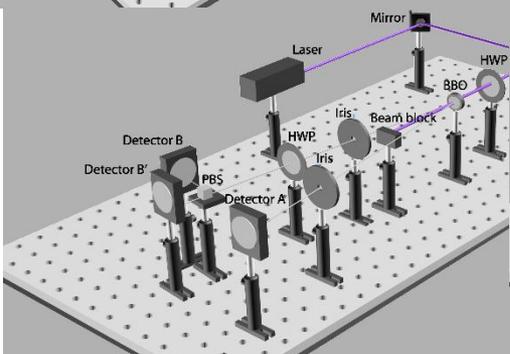
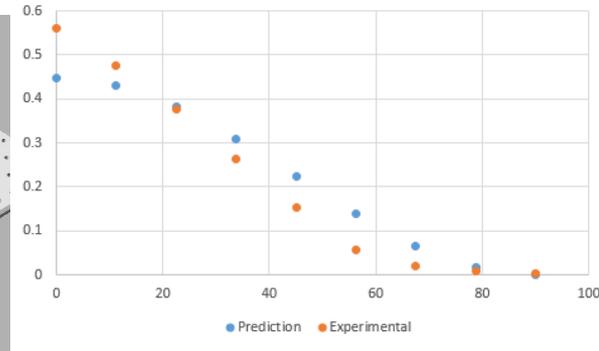
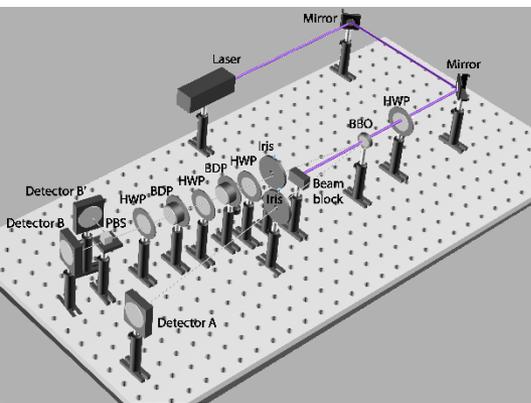


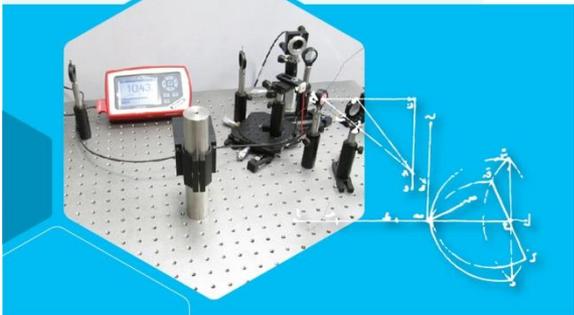
Single Photon Quantum Information Experiments Any Physics Department Can Build!

Muhammad Sabieh Anwar






گوشہ ابن سہل
IBN SAHL
 Corner for Optics
 روشنی کی تجربہ گاہ



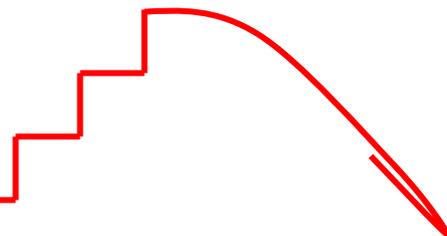
ابن سہل گوشہ کے لیے
 سائنس اور انجینئرنگ، لومس (LUMS)، لاہور
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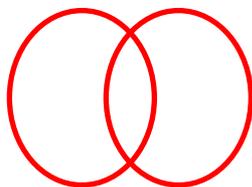
Salient Features of a Quantum Understanding of the World

- Quantization

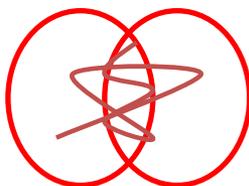


- Superposition

- Entanglement

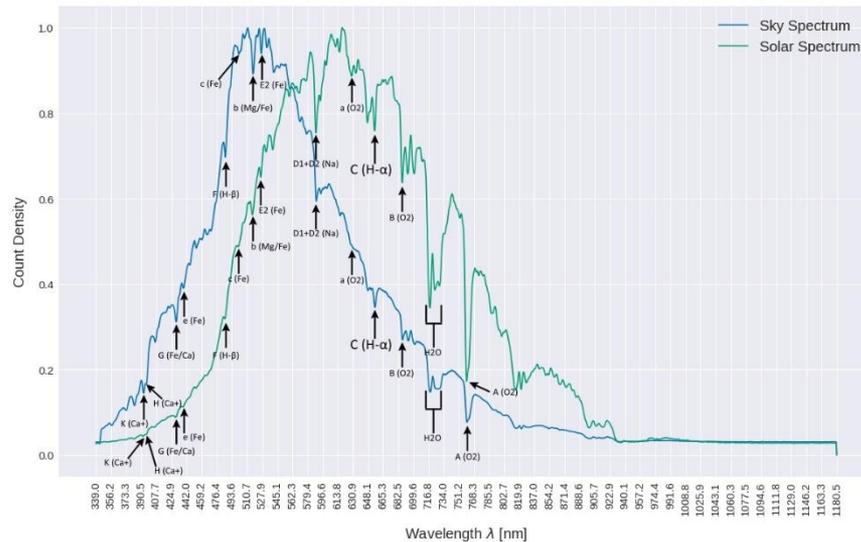
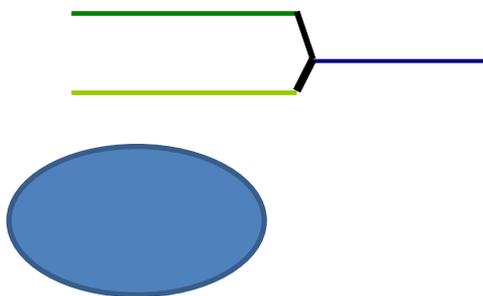


- Interference



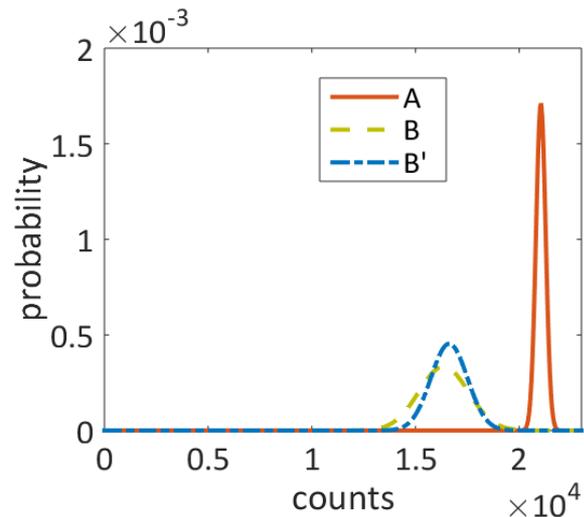
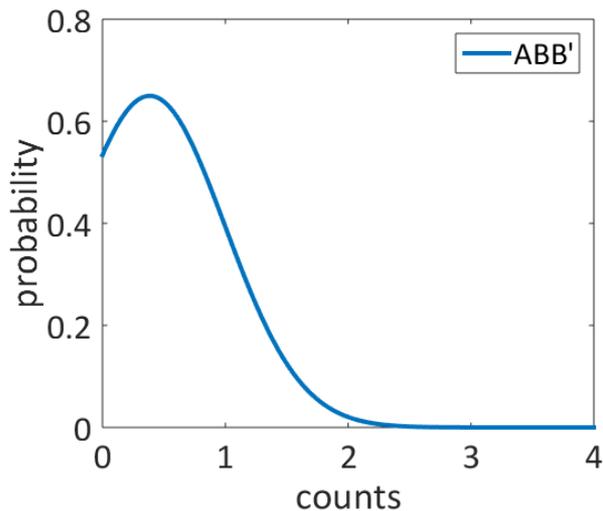
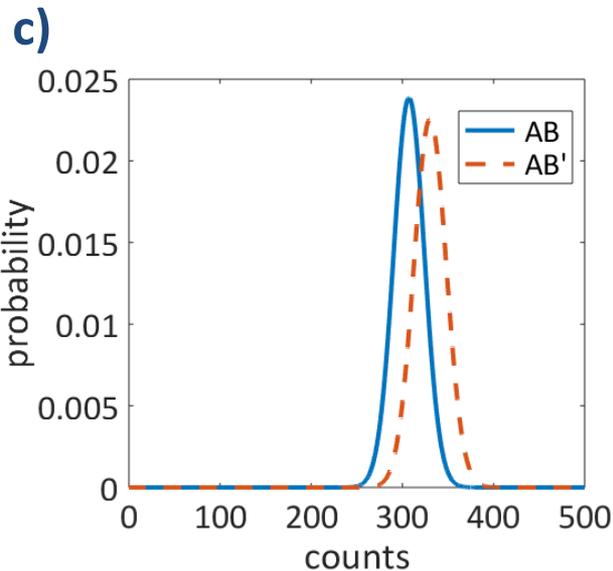
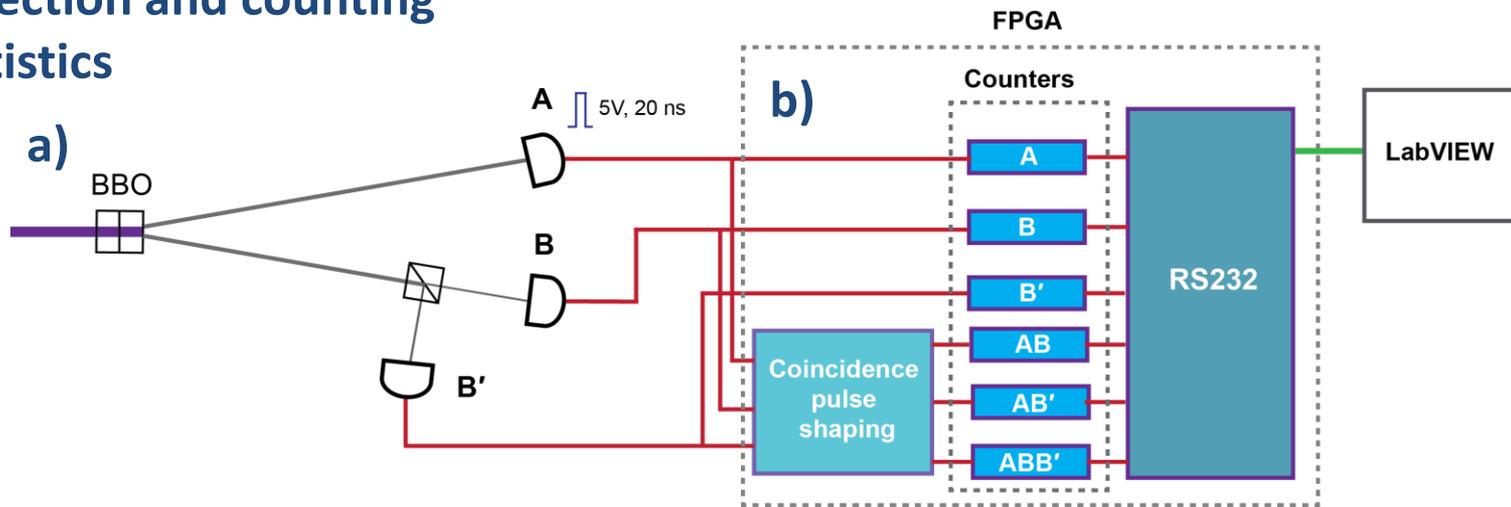
- Uncertainty

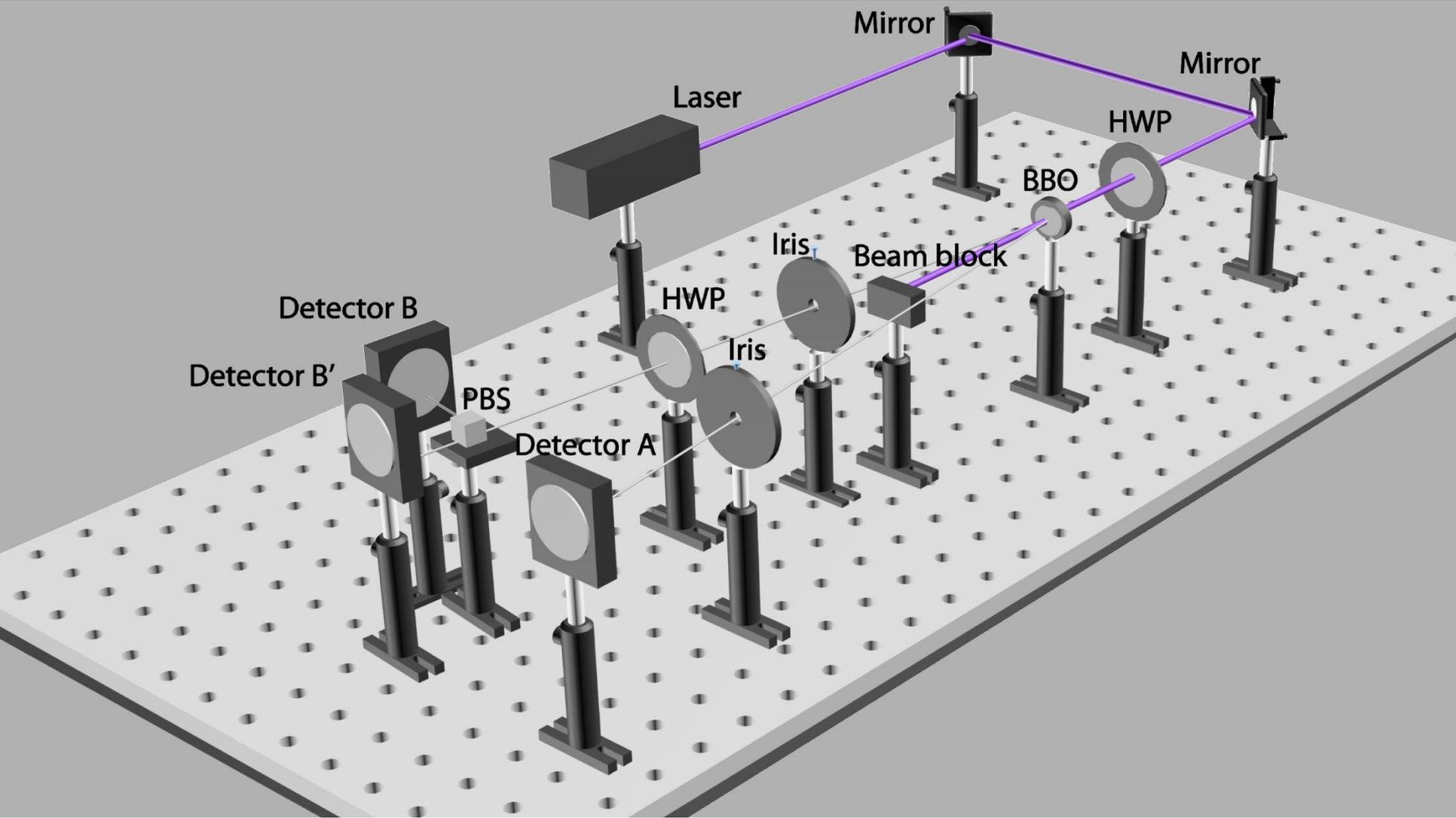
- Measurement

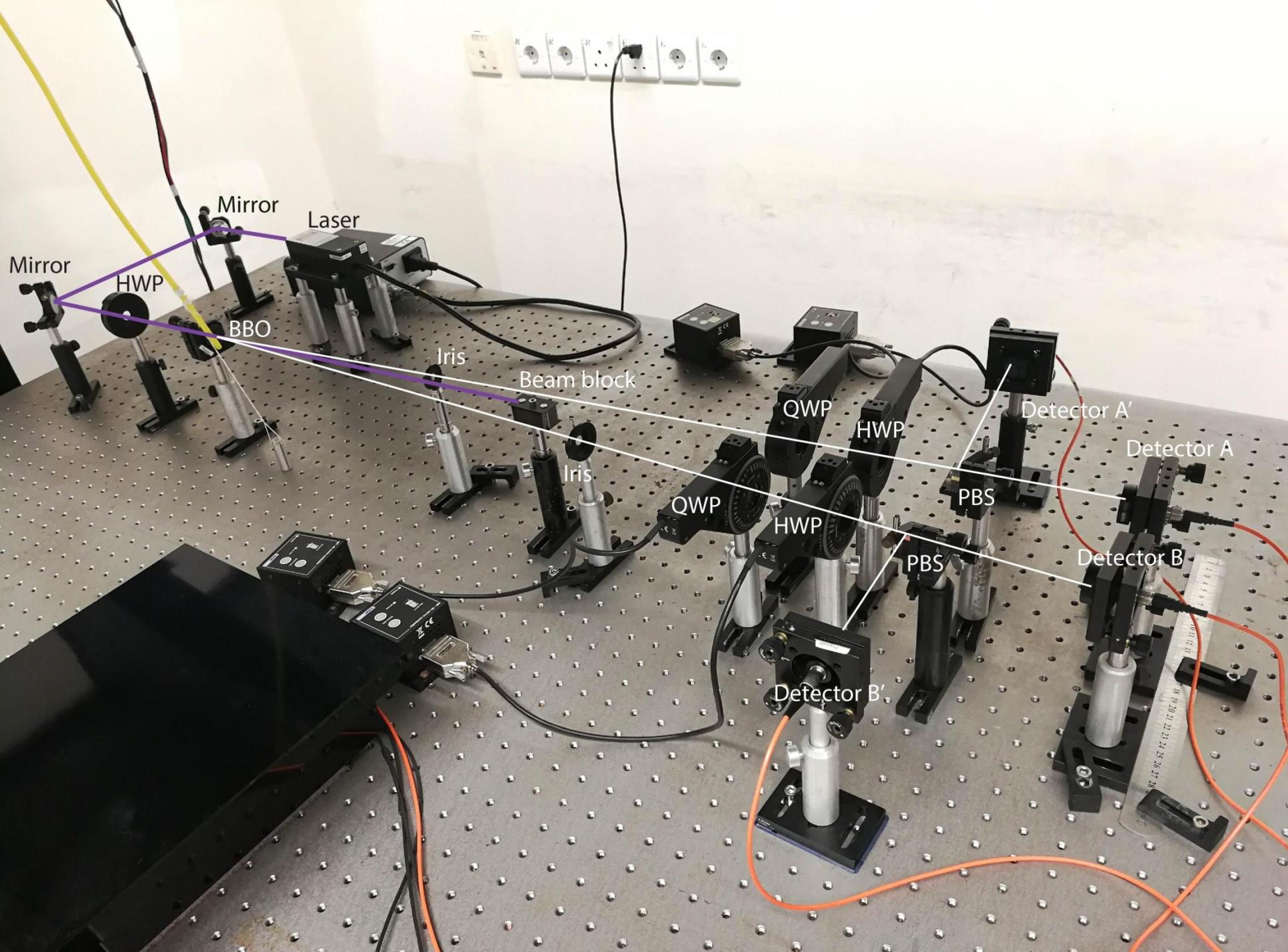


Overview of the single photon lab

- a) Optical setup
- b) Photon detection and counting
- c) Photon statistics







Mirror

Laser

Mirror

HWP

BBO

Iris

Beam block

QWP

HWP

Detector A'

Detector A

Iris

QWP

HWP

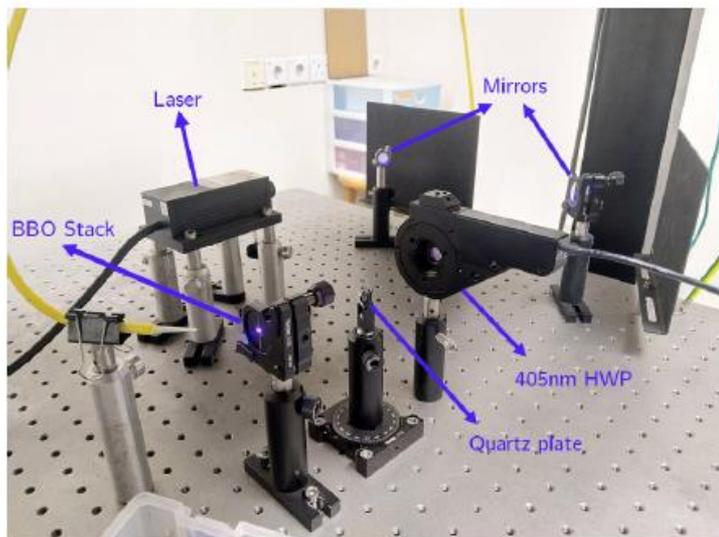
PBS

Detector B

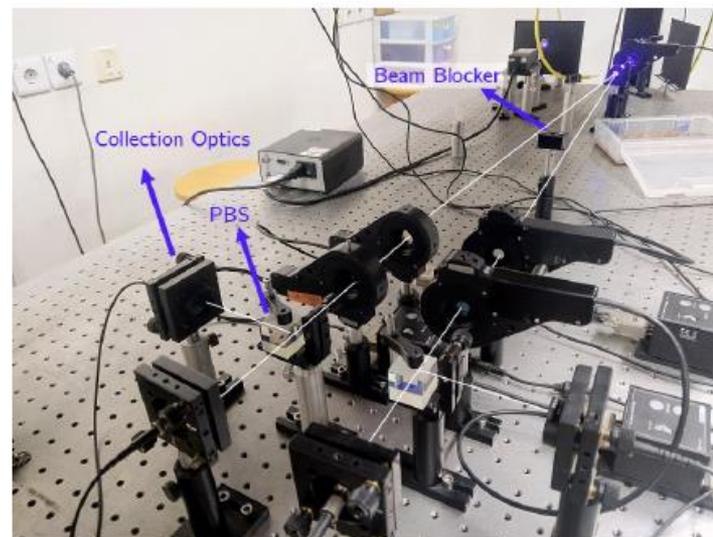
PBS

Detector B'





(a) Photon generation.



(b) Photon collection.

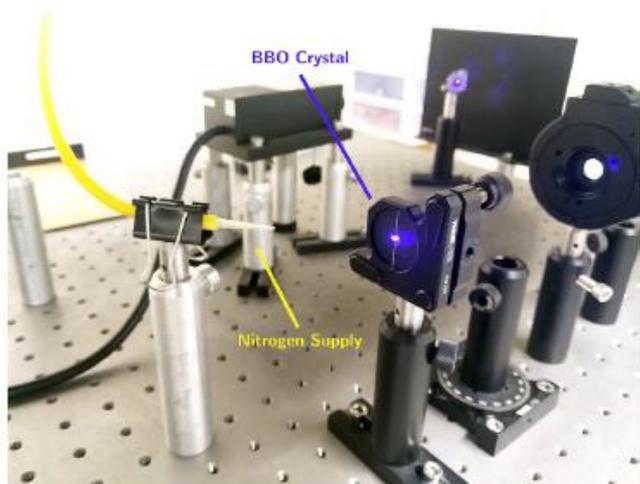
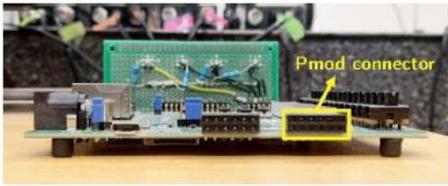


Figure 1.4: 405 nm pump beam hitting the BBO crystal stack. BBO crystals down-convert 405nm photons into two 810nm photons.



Figure 1.6: An SPCM on the left and its power supply on the right.



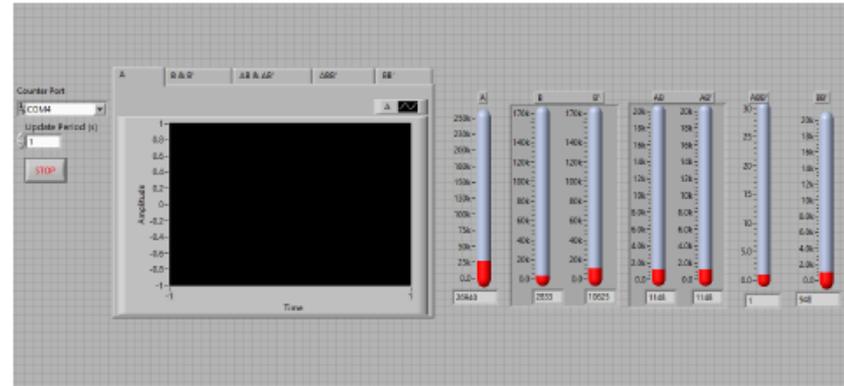
(a) Pmod connector on a Nexys A7.



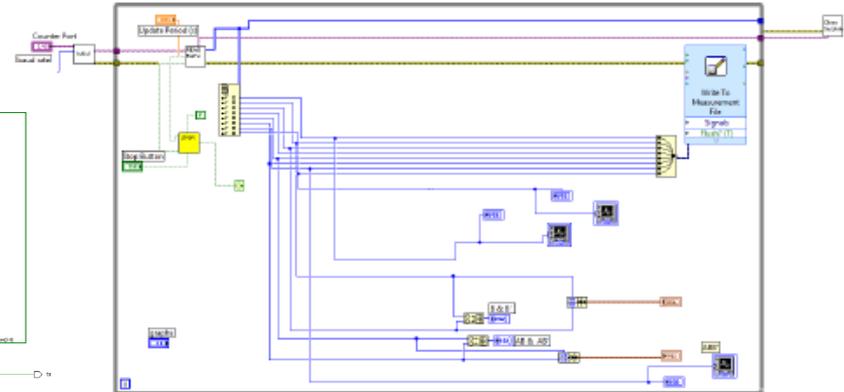
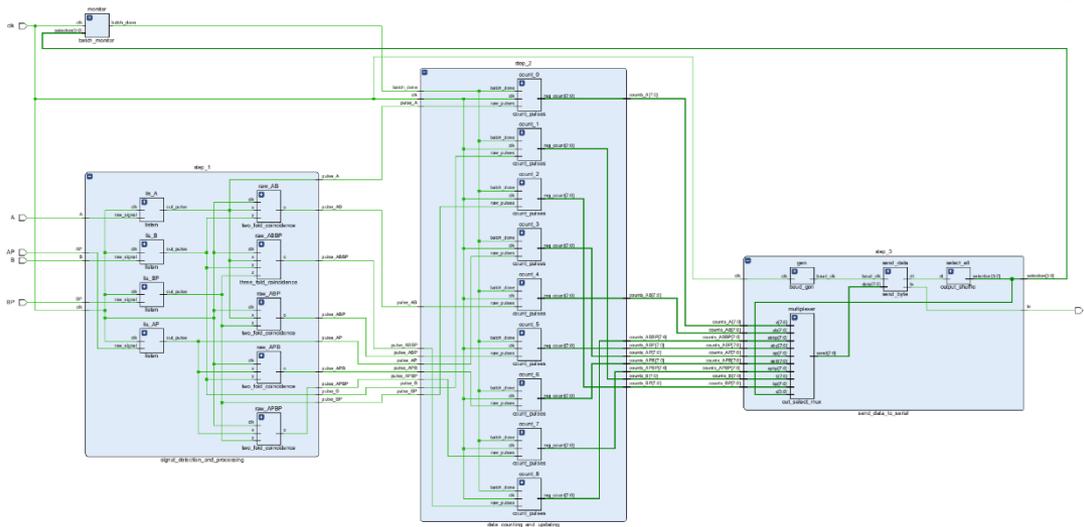
(b) UART serial communication device



(c) Nexys A7 FPGA.



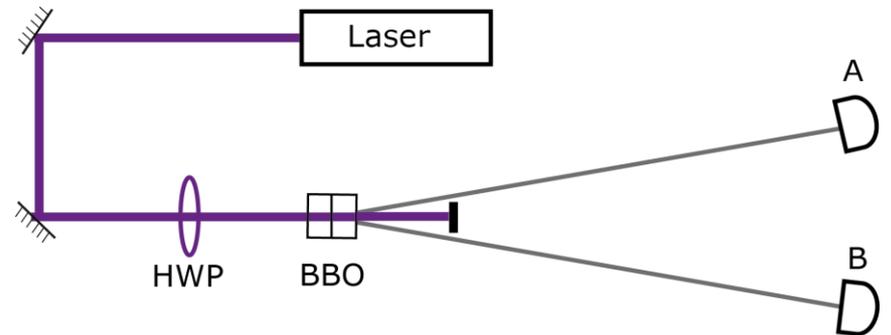
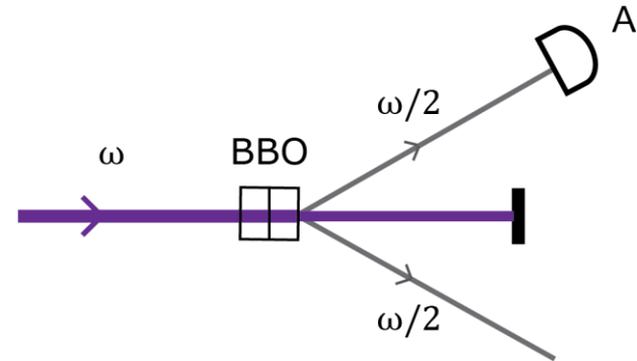
(a)



(b)

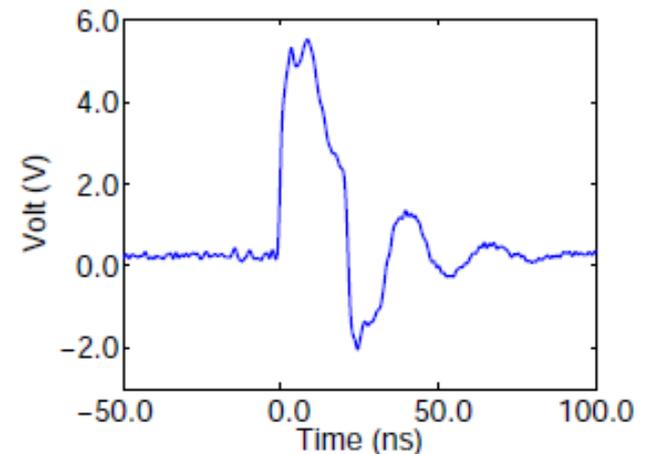
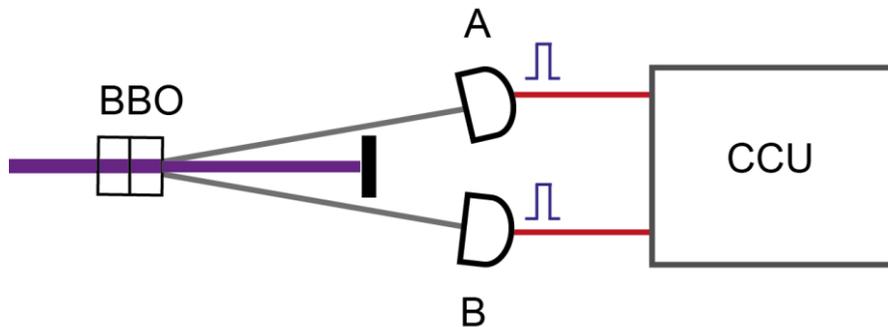
Producing single photons

- Type I spontaneous parametric downconversion with β -barium borate (BBO)¹
- Pump beam 405 nm, 50 mW
- Generation of polarization-entangled photon beams
- Gating of one beam at detector A projects the other beam into single photon state.



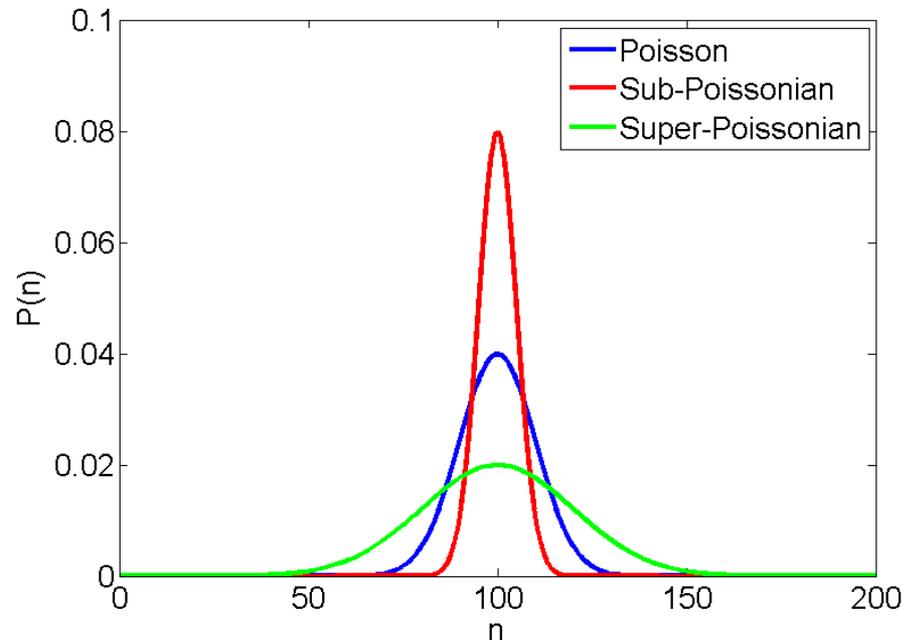
Detecting single photons

- Each detector comprises an avalanche photodiode that outputs a 20 ns 5V pulse on each detection of a photon.
- Pulses coincident at two or more detectors are counted by a coincidence counting unit (CCU).
- Pulses from individual detectors are also counted by the CCU.
- The counts are transmitted to a computer for visualization and analysis of results.



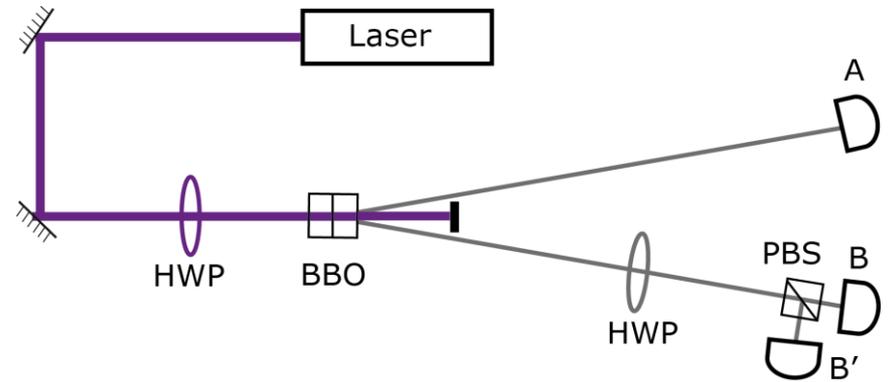
What do photon statistics tell us?

- Classically, sub-Poissonian ($\Delta n < \sqrt{\bar{n}}$) light is impossible.
- A coherent source shows Poissonian ($\Delta n = \sqrt{\bar{n}}$) whereas a fluctuating source shows super-Poissonian ($\Delta n > \sqrt{\bar{n}}$) statistics.
- Our single-detector counts show super-Poissonian while the gated (coincidence) counts show sub-Poissonian statistics.



Particle nature of light

- Hypothesis: a single particle can only be detected at one place⁴.



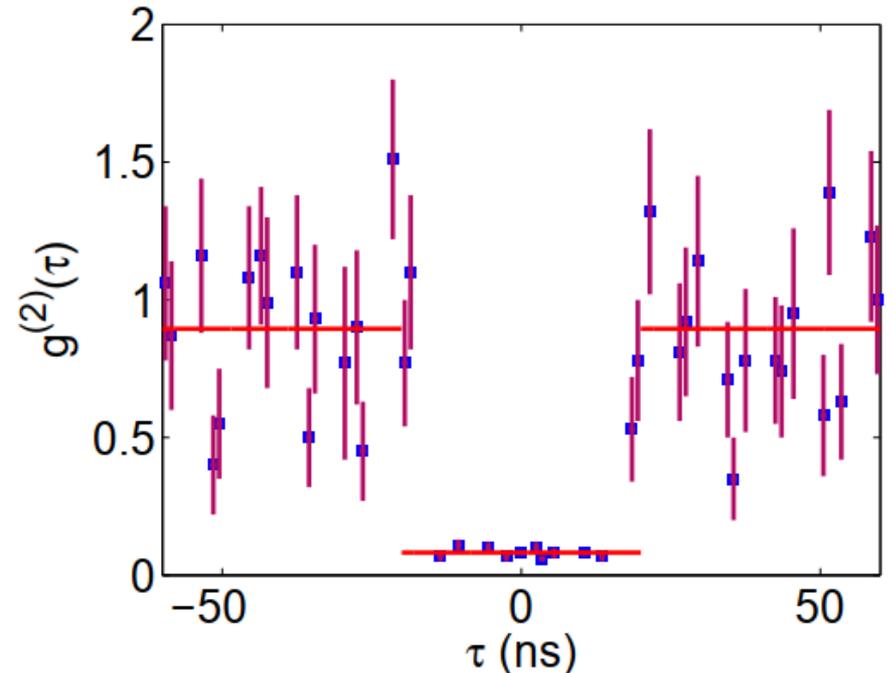
- 2nd order correlation function

$$g^{(2)}(0) = \frac{P_{ABB'}}{P_{AB}P_{AB'}}$$

- In an experimental run of 10 minutes, we obtained

$$g^{(2)}(0) = 0.080 \pm 0.005$$

- Hanbury Brown-Twiss experiment



Measuring the quantum state - I

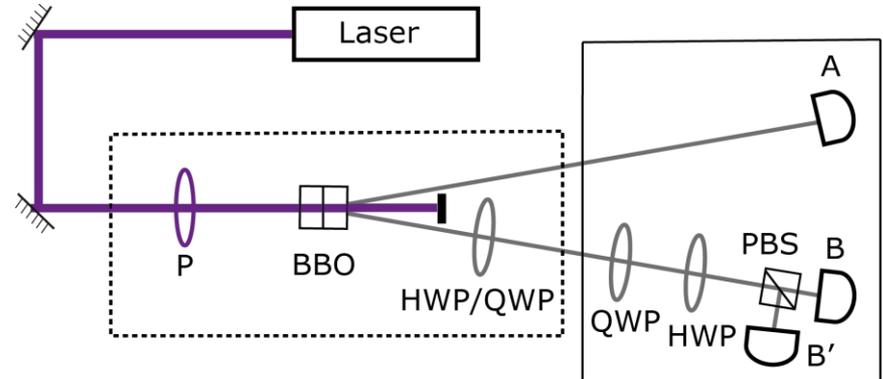
- A general pure state can be expressed as

$$|\Psi\rangle = a |H\rangle + be^{i\phi} |V\rangle$$

- Measurement destroys the

single photon. So we perform measurements on identically prepared photons.

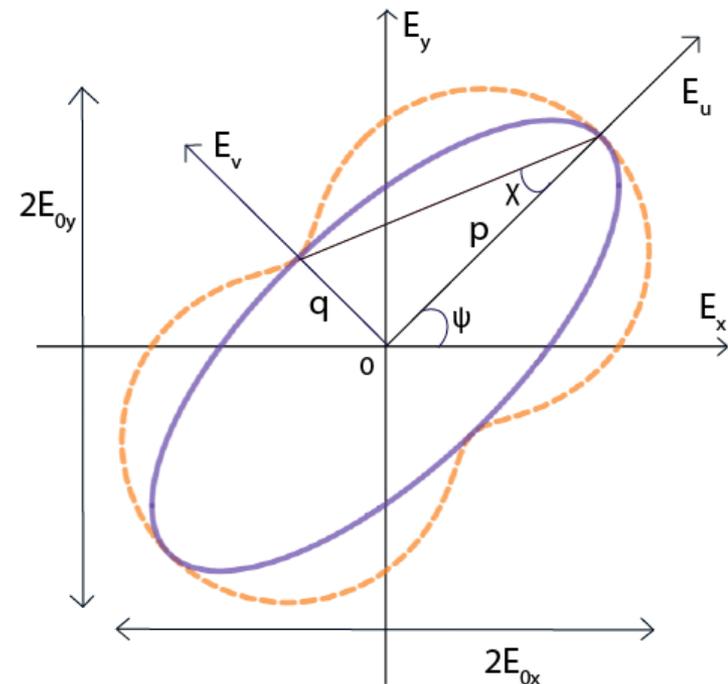
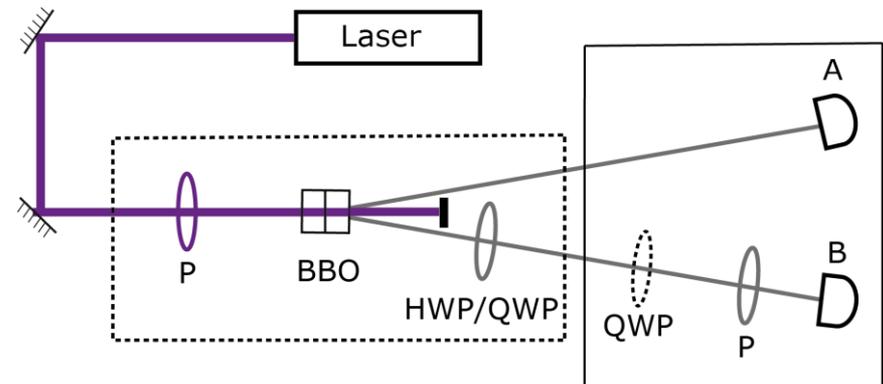
- Three parameters require us to make measurements in three different bases.

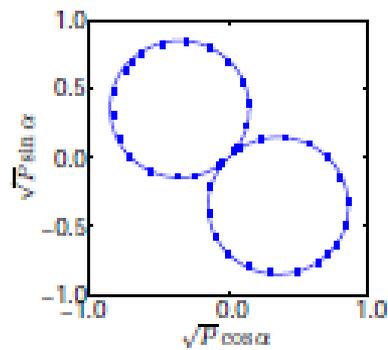


Input	Prediction	Measurement
$ H\rangle$	$a = 1.000, b = 0.000$	$a = 0.986(2), b = 0.165(2),$ $\theta = -1.450(5)$
$ 45^\circ\rangle$	$a = 0.707, b = 0.707,$ $\theta = 0.000$	$a = 0.743(2), b = 0.669(2),$ $\theta = 0.184(2)$
$ L\rangle$	$a = 0.707, b = 0.707,$ $\theta = -1.571$	$a = 0.663(2), b = 0.749(2),$ $\theta = -1.539(5)$

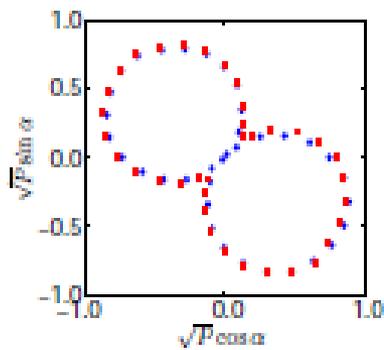
Measuring the quantum state - II

- The peanut method
- Used in antenna polarimetry, optical polarimetry, quantum state measurement
- Applies both to classical and quantum light (single qubit)
- Can be used to determine the Stokes parameters or the ellipse parameters

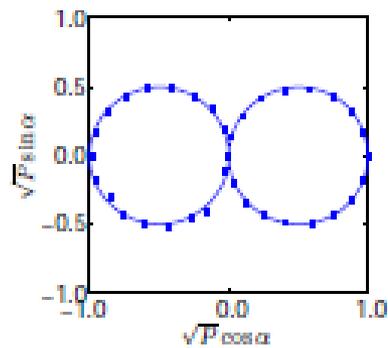




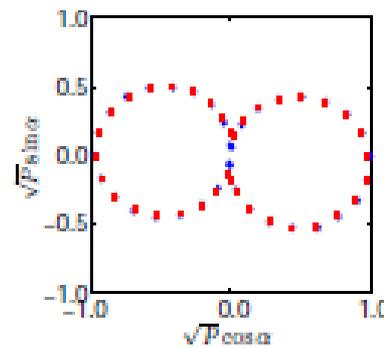
(a)



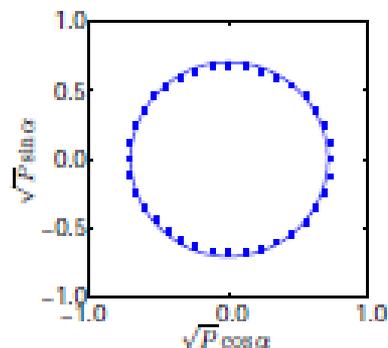
(b)



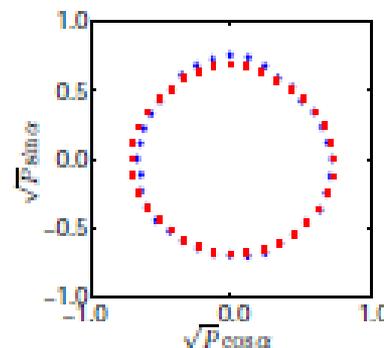
(c)



(d)



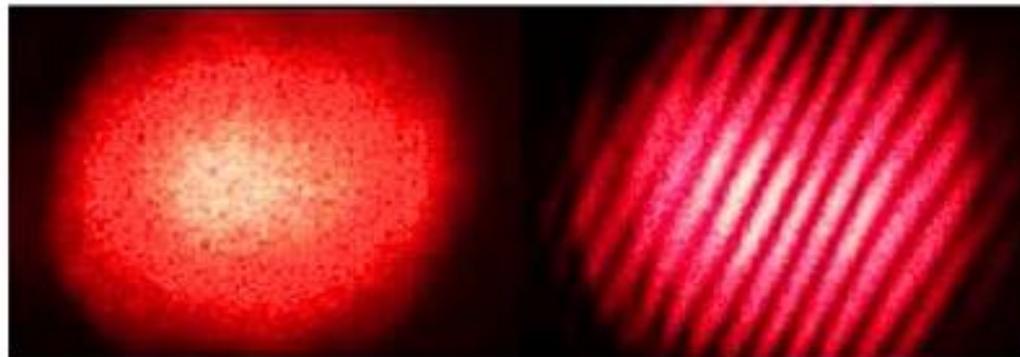
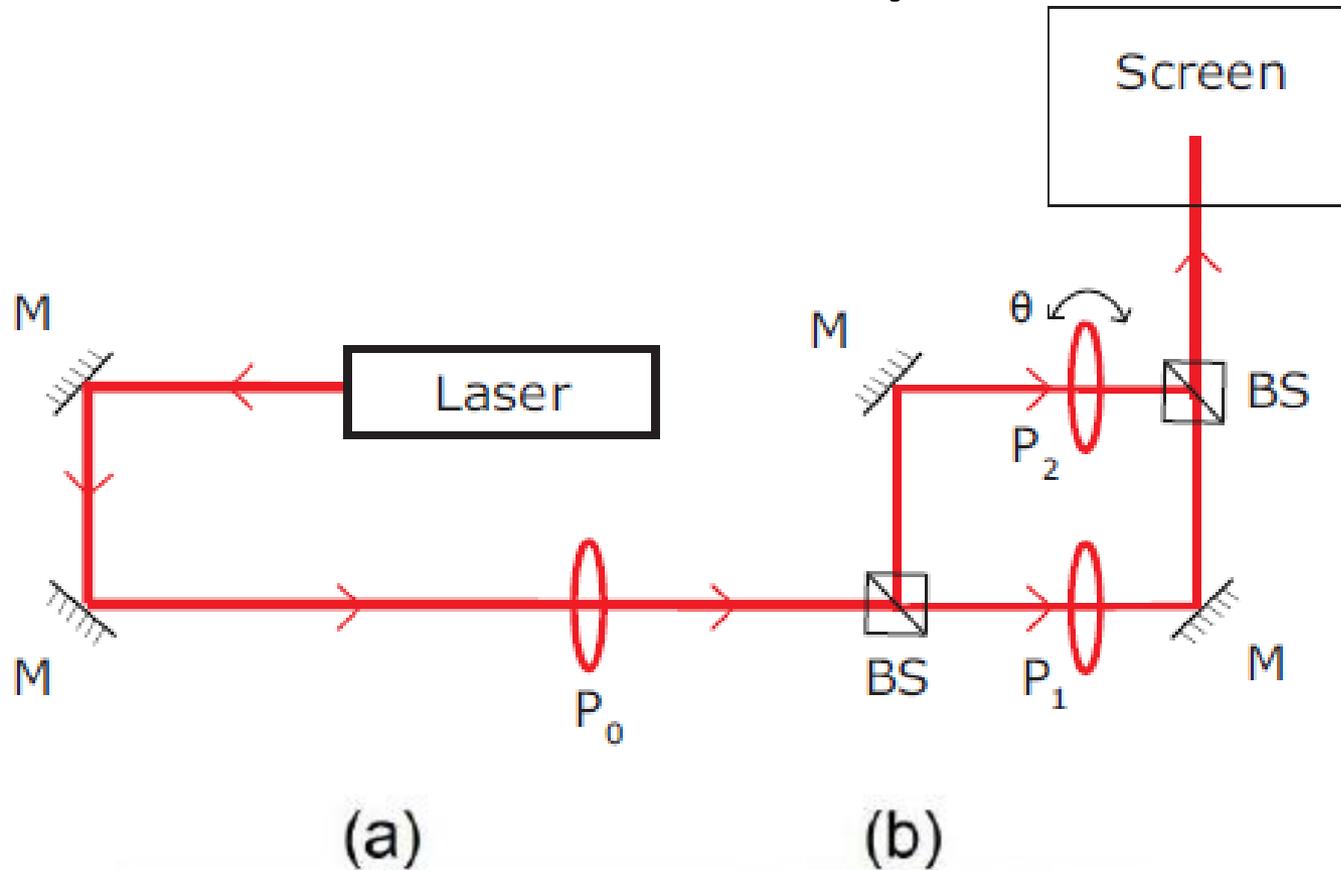
(e)

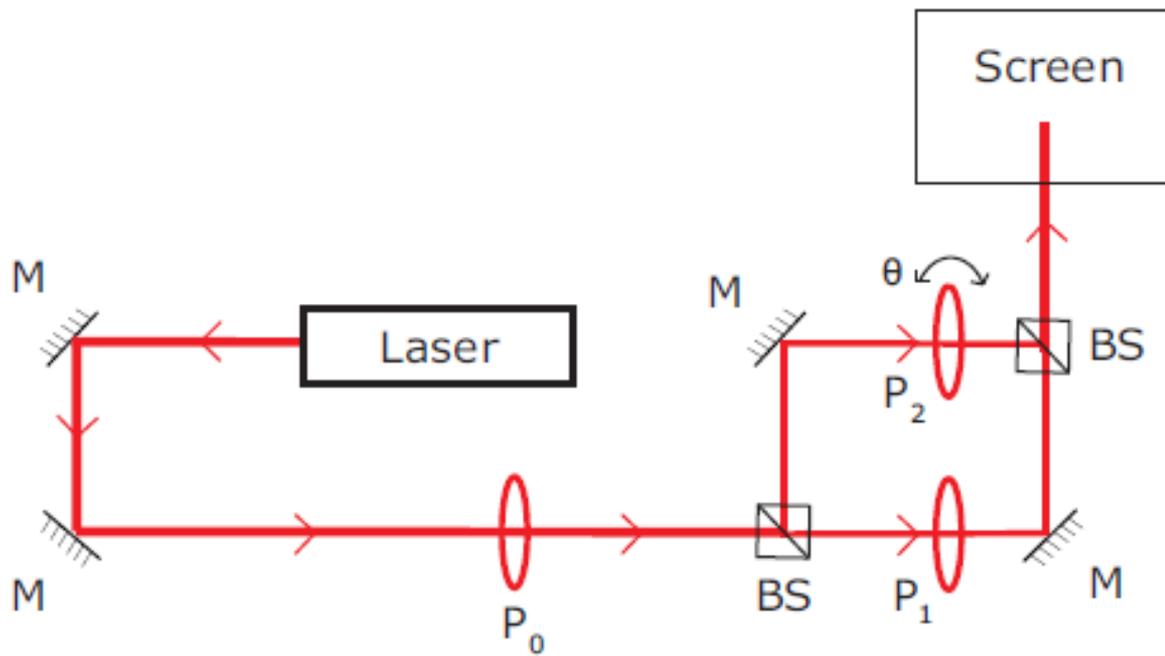


(f)

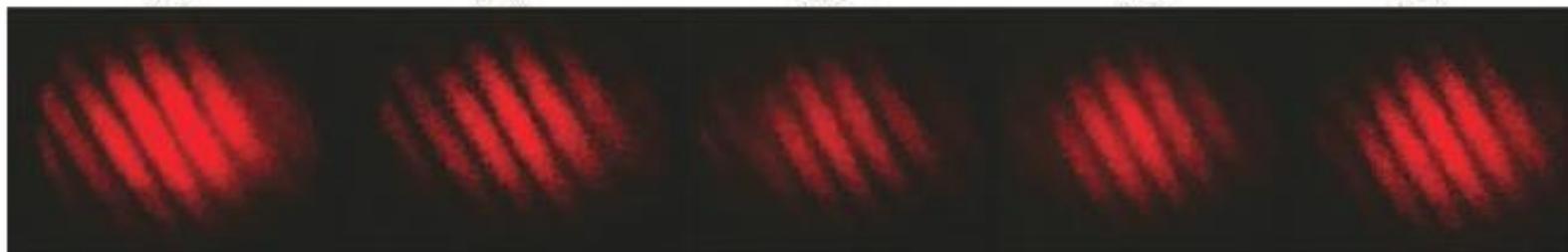
Polarization state	One-shot measurement	Quantum hippopede	Classical hippopede
Horizontal	$S_0 = 1.00 \pm 0.00$	1.00 ± 0.04	1.00 ± 0.04
	$S_1 = 0.94 \pm 0.00$	0.95 ± 0.04	0.92 ± 0.04
	$S_2 = 0.04 \pm 0.00$	-0.16 ± 0.04	-0.15 ± 0.04
Vertical	$S_0 = 1.00 \pm 0.00$	1.00 ± 0.02	1.00 ± 0.02
	$S_1 = -0.86 \pm 0.00$	-0.98 ± 0.02	-0.92 ± 0.02
	$S_2 = 0.20 \pm 0.00$	-0.08 ± 0.02	-0.09 ± 0.02
Anti-diagonal	$S_0 = 1.00 \pm 0.02$	1.00 ± 0.00	1.00 ± 0.03
	$S_1 = -0.03 \pm 0.00$	0.11 ± 0.02	0.13 ± 0.03
	$S_2 = -1.00 \pm 0.00$	-0.97 ± 0.02	-0.93 ± 0.03
Diagonal	$S_0 = 1.00 \pm 0.00$	1.00 ± 0.00	1.00 ± 0.00
	$S_1 = 0.10 \pm 0.00$	0.04 ± 0.02	0.04 ± 0.02
	$S_2 = 0.98 \pm 0.00$	1.00 ± 0.00	1.00 ± 0.00
Left circular	$S_0 = 1.00 \pm 0.00$	1.00 ± 0.04	1.00 ± 0.02
	$S_1 = -0.12 \pm 0.00$	-0.04 ± 0.04	0.06 ± 0.02
	$S_2 = 0.03 \pm 0.01$	-0.02 ± 0.04	-0.03 ± 0.02
Right circular	$S_0 = 1.00 \pm 0.00$	1.00 ± 0.04	1.00 ± 0.02
	$S_1 = 0.07 \pm 0.00$	-0.07 ± 0.04	0.06 ± 0.02
	$S_2 = 0.09 \pm 0.00$	-0.04 ± 0.06	-0.04 ± 0.02

Classical Erasure Experiment

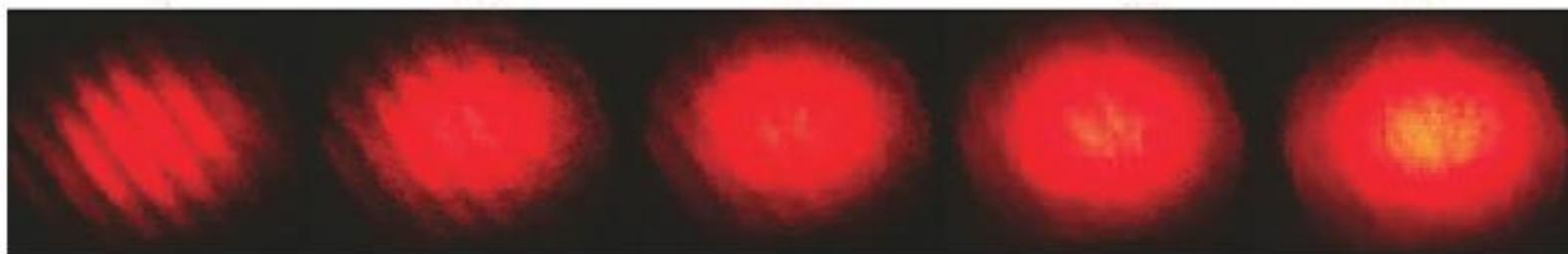


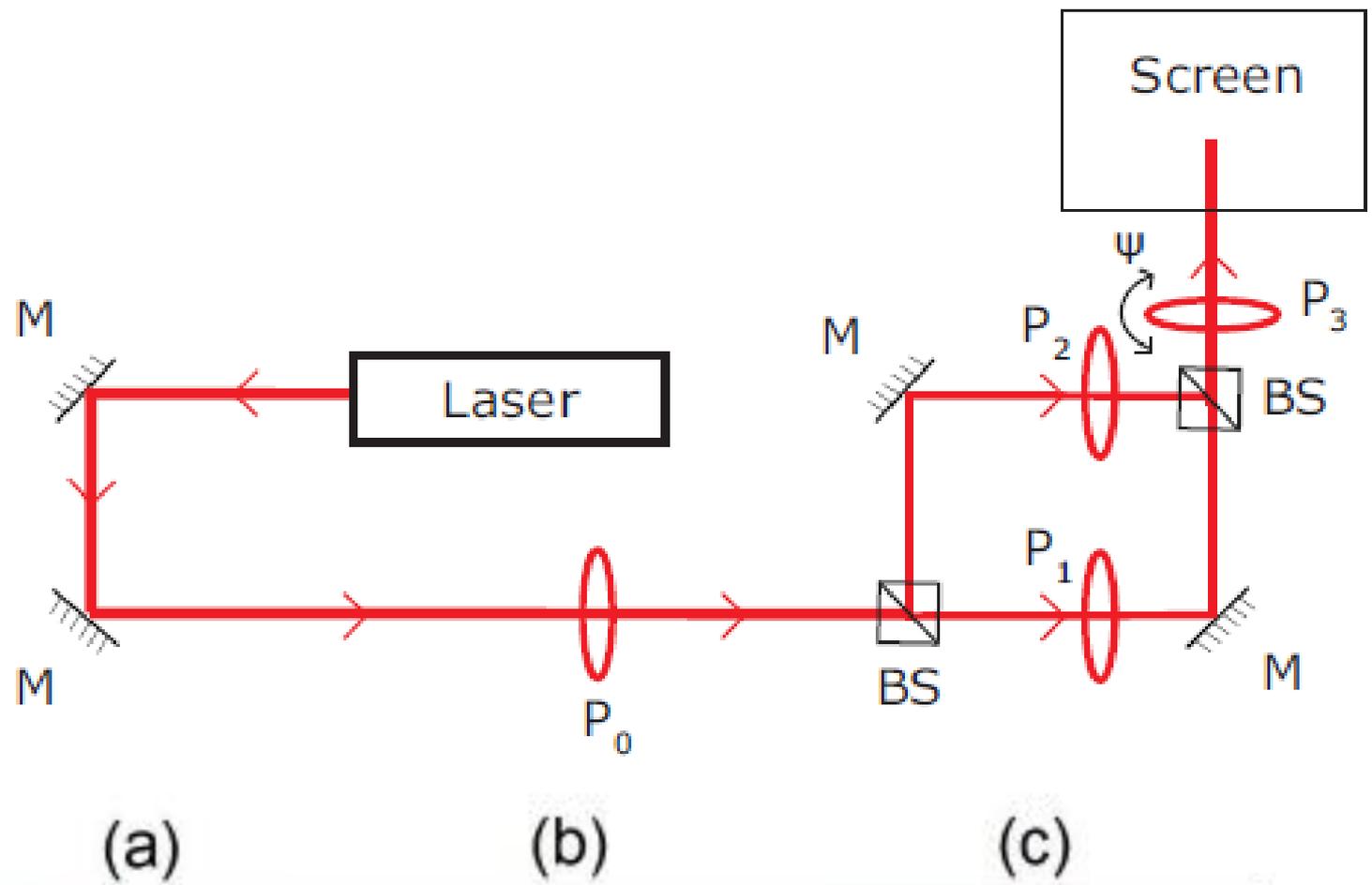


(a) (b) (c) (d) (e)

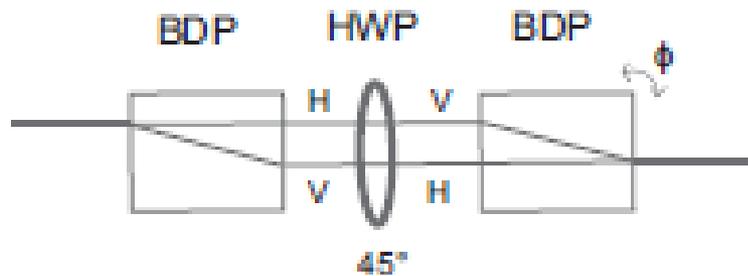
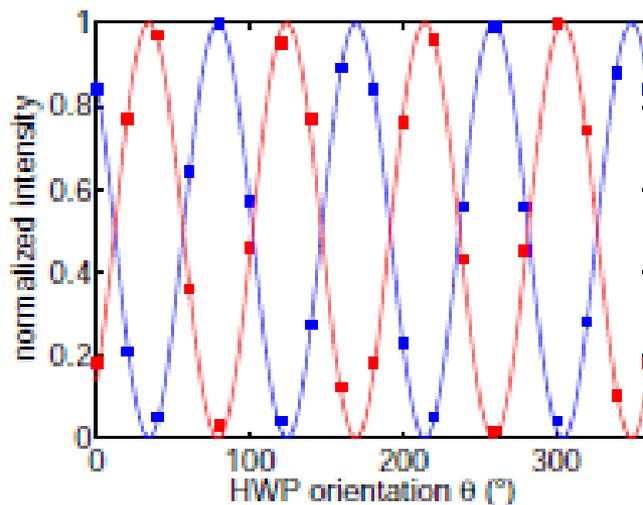
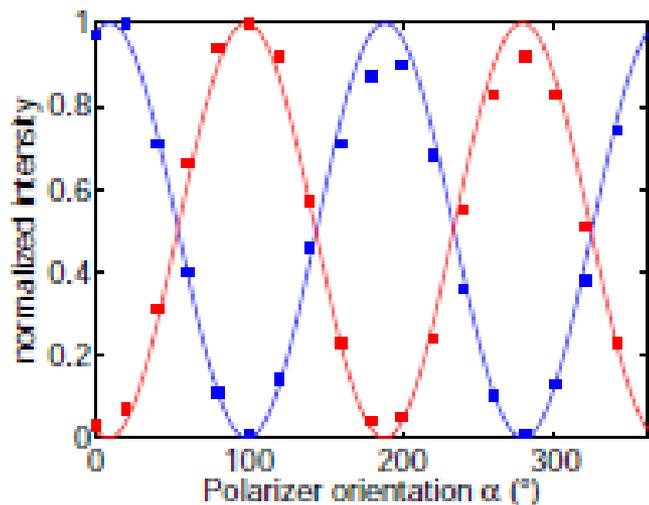
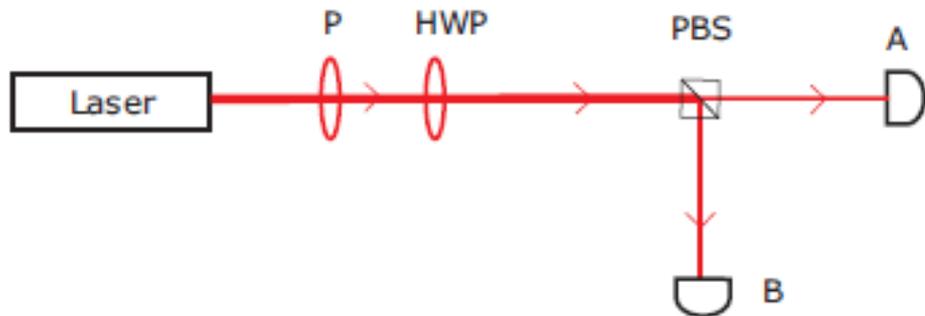


(f) (g) (h) (i) (j)

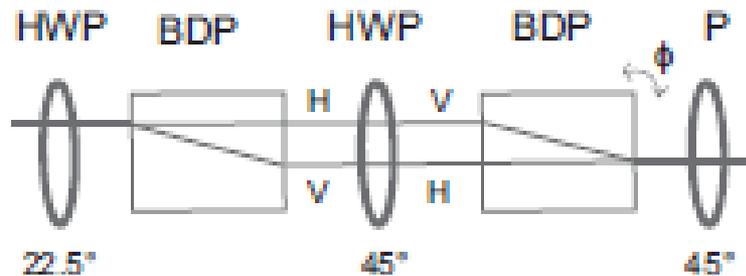




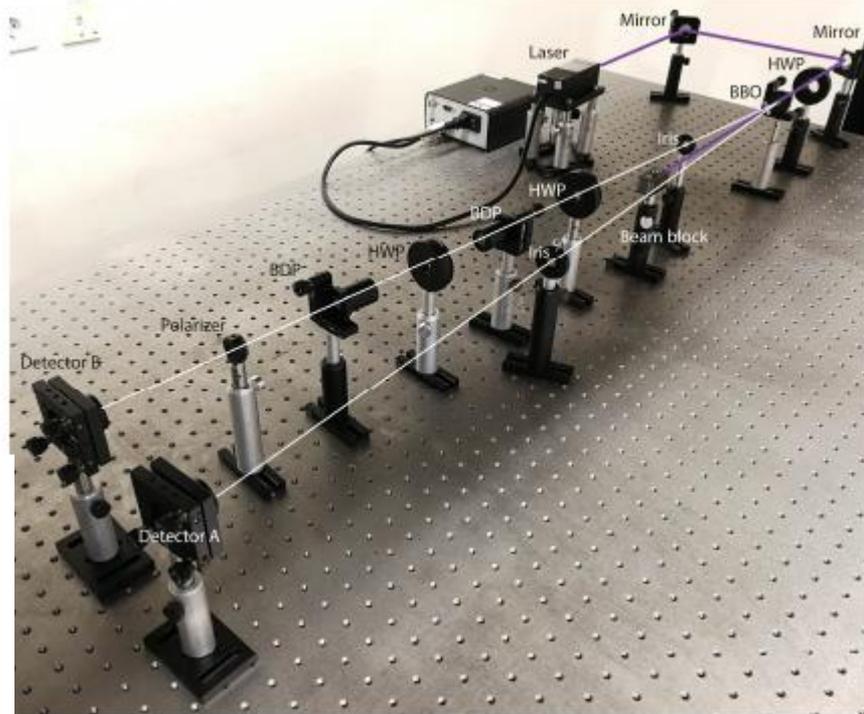
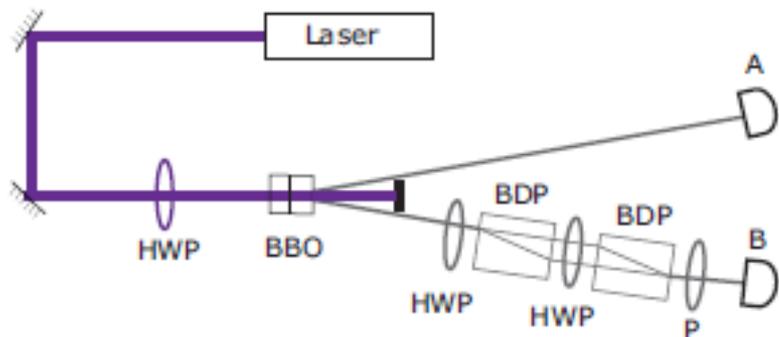
Interferometry and Quantum Erasure



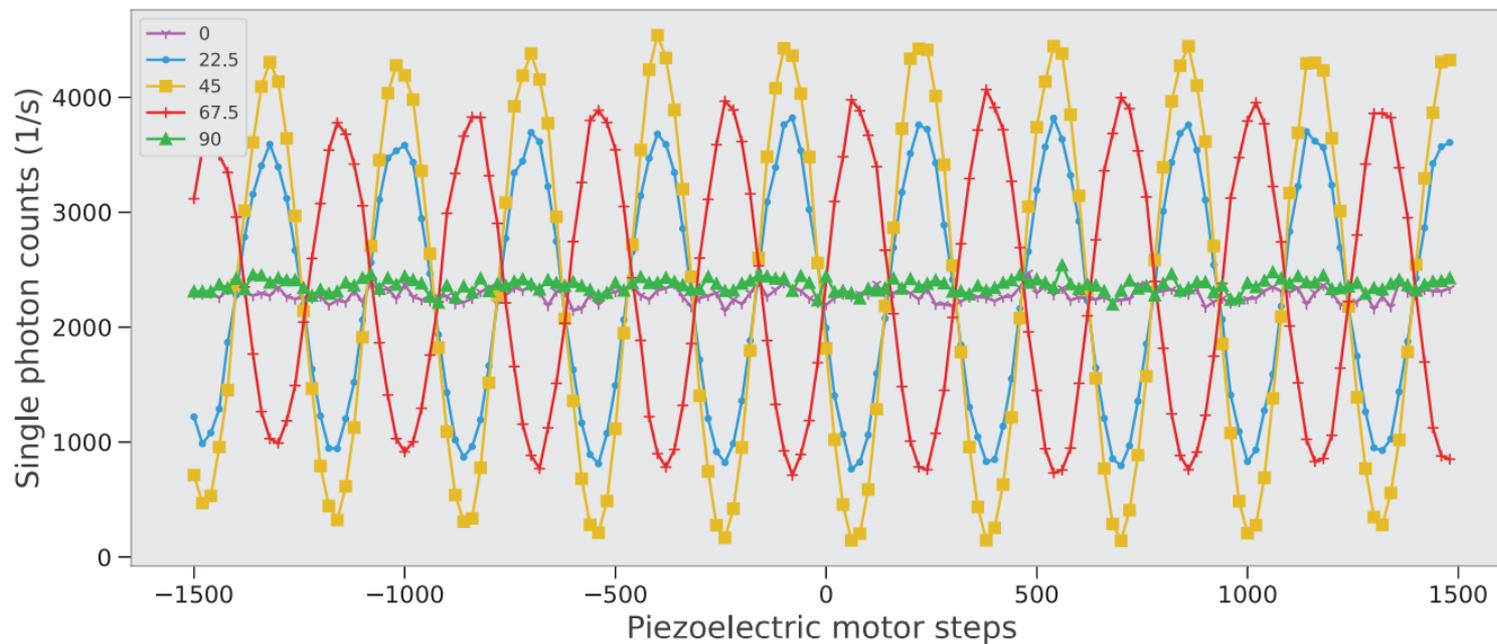
(a)



(b)

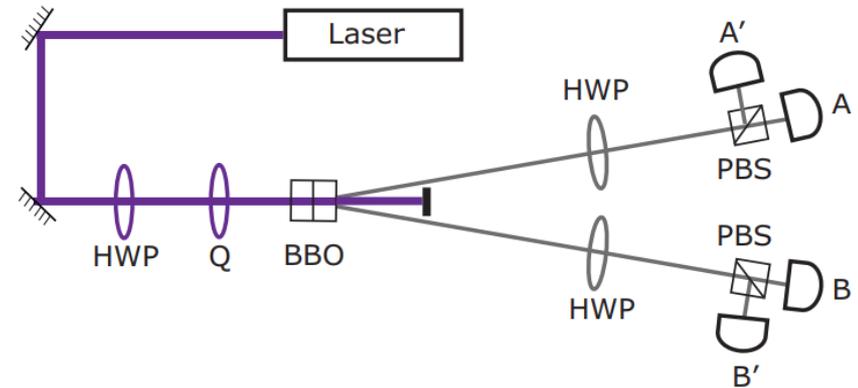


Polarizer orientation (°)	Measured Visibility	Predicted Visibility
0.0	0.08	0.00
22.5	0.56	0.71
45.0	0.84	1.00
67.5	0.58	0.71
90.0	0.11	0.00



Testing entanglement – Nonlocality I

- Entanglement gives rise to ‘spooky action at a distance’. There could be a hidden variable involved.



- Bell inequalities – experimental tests for local realistic theories:

- CHSH test of local realism⁷

$$|S| \leq 2$$

$$S = E(a, b) - E(a, b') + E(a', b) + E(a', b')$$

$$E(\alpha, \beta) = P_{HH} + P_{VV} - P_{HV} - P_{VH}$$

- We prepared the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|H\rangle_A |H\rangle_B + |V\rangle_A |V\rangle_B)$$

and for the angles

$$a = -45^\circ, a' = 0^\circ,$$

$$b = -22.5^\circ, b' = 22.5^\circ$$

we obtained

$$S = 2.331 \pm 0.004$$

Testing entanglement – Nonlocality II

- Hardy's test of local realism
- Another test – another inequality

$$H \leq 0$$

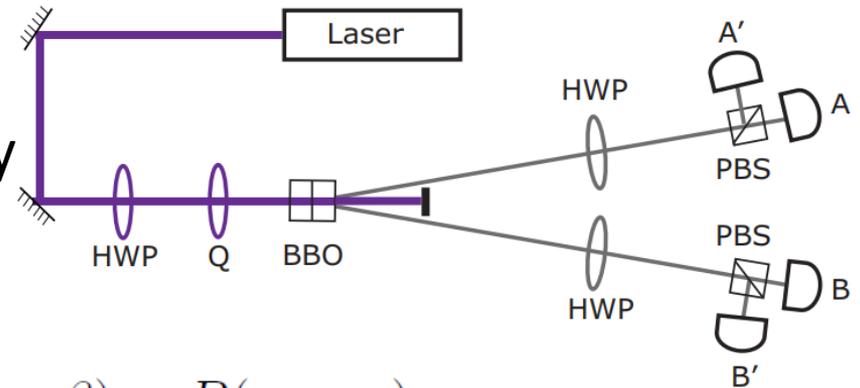
$$H = P(\beta, -\beta) - P(\beta, \alpha^\perp) - P(-\alpha^\perp, -\beta) - P(-\alpha, \alpha)$$

- We created the state

$$|\psi\rangle = \sqrt{0.8} |H\rangle_A |H\rangle_B + \sqrt{0.2} |V\rangle_A |V\rangle_B$$

and obtained

$$H = 0.107 \pm 0.002 \quad \text{which violates local realism.}$$



Testing entanglement – Nonlocality III

- Freedman's test of local realism
- Yet another inequality!

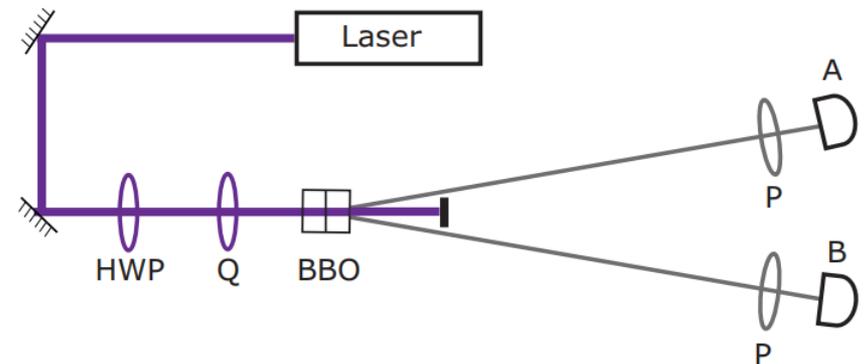
$$\delta = \left| \frac{N(22.5^\circ) - (67.5^\circ)}{N_0} \right| - \frac{1}{4} \leq 0$$

- We created the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|H\rangle_A |H\rangle_B + |V\rangle_A |V\rangle_B)$$

and obtained

$\delta = 0.095 \pm 0.003$ which violates local realism.



How do we check if the state is perfectly entangled?

Gauging entanglement? Tomography!

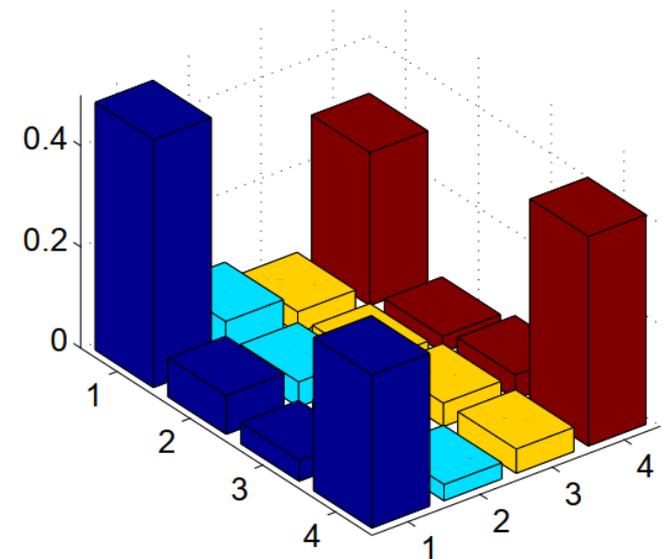
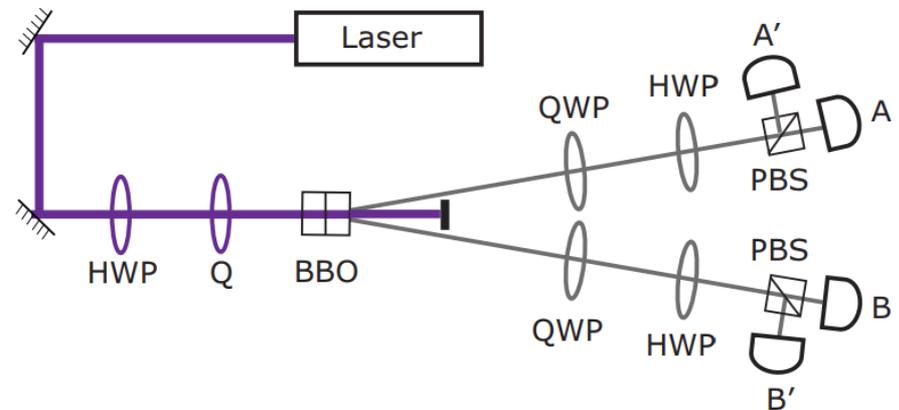
- A set of measurements with different bases to determine the two-qubit density matrix⁹:

$$\hat{\rho} = \frac{1}{4} \sum_{i,j=0}^3 S_{ij} \hat{\sigma}_i \otimes \hat{\sigma}_j$$

- For the Bell state

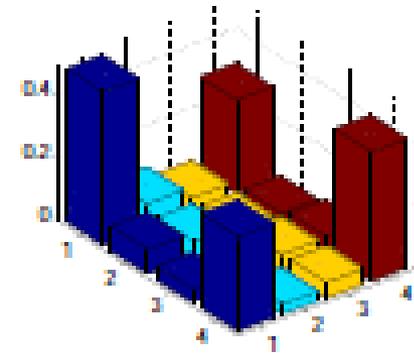
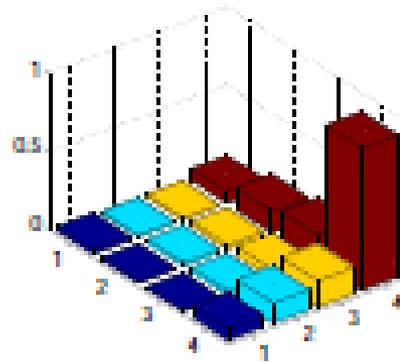
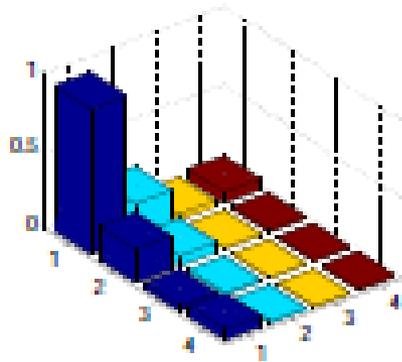
$$|\psi\rangle = \frac{1}{\sqrt{2}}(|H\rangle_A |H\rangle_B + |V\rangle_A |V\rangle_B)$$

we obtained the density matrix with fidelity 0.75 using maximum likelihood estimation⁹.



State	Predicted density matrix	Measured density matrix
$ H\rangle$	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0.97 & -0.04 - 0.31i \\ -0.04 + 0.31i & 0.03 \end{pmatrix}$
$ V\rangle$	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0.07 & 0.10 + 0.22i \\ 0.10 - 0.22i & 0.93 \end{pmatrix}$
$ D\rangle$	$\begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.55 & 0.32 - 0.17i \\ 0.32 + 0.17i & 0.45 \end{pmatrix}$
$ A\rangle$	$\begin{pmatrix} 0.5 & -0.5 \\ -0.5 & 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.49 & -0.45 - 0.04i \\ -0.45 + 0.04i & 0.51 \end{pmatrix}$
$ L\rangle$	$\begin{pmatrix} 0.5 & 0.5i \\ -0.5i & 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.44 & 0.01 + 0.43i \\ 0.01 - 0.43i & 0.56 \end{pmatrix}$
$ R\rangle$	$\begin{pmatrix} 0.5 & -0.5i \\ 0.5i & 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.53 & 0.04 - 0.47i \\ 0.04 + 0.47i & 0.47 \end{pmatrix}$

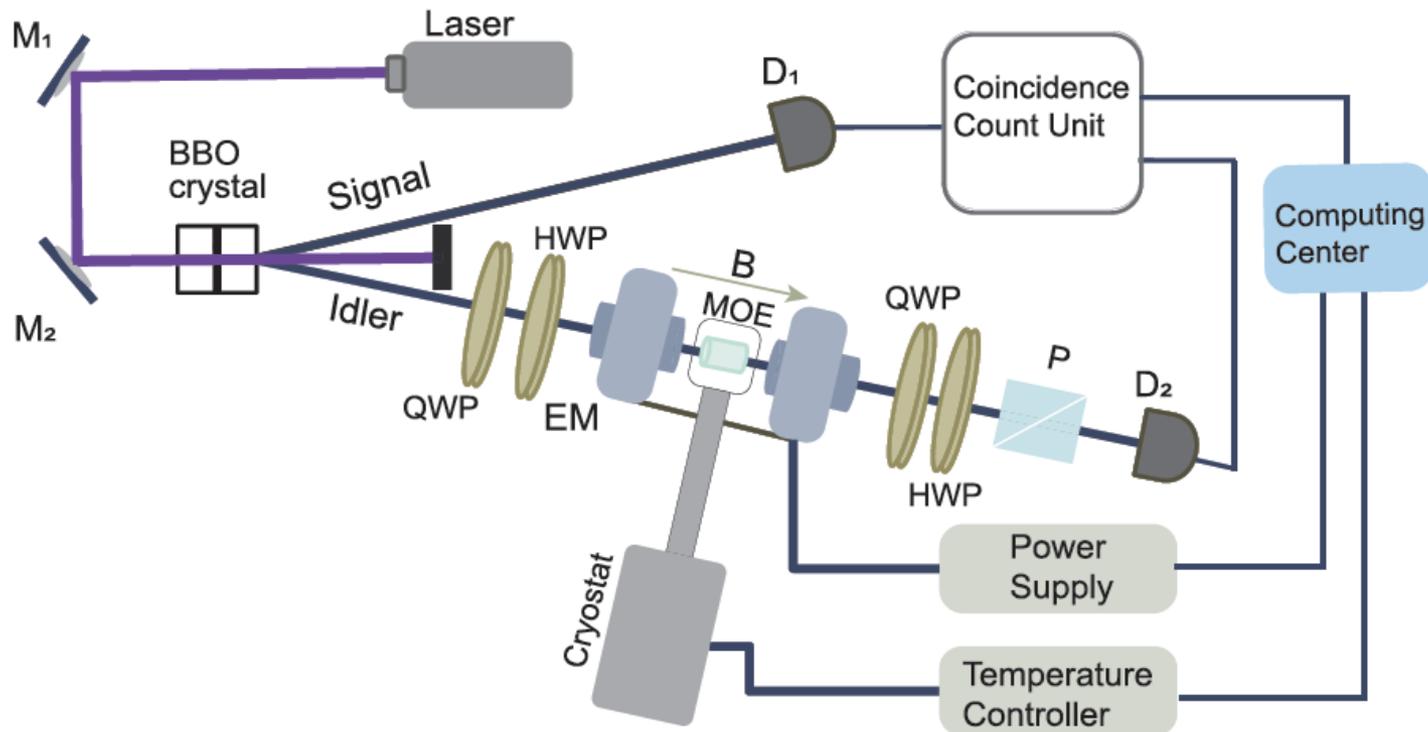
Table 6.1: Results of single qubit tomography.



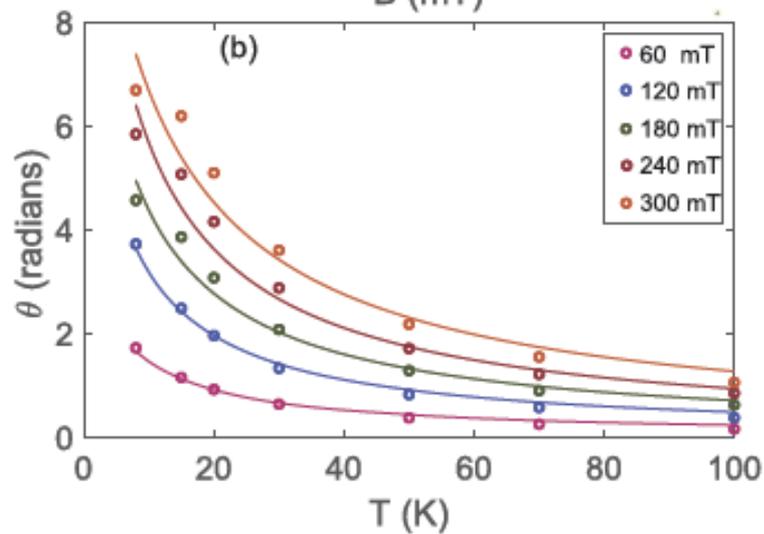
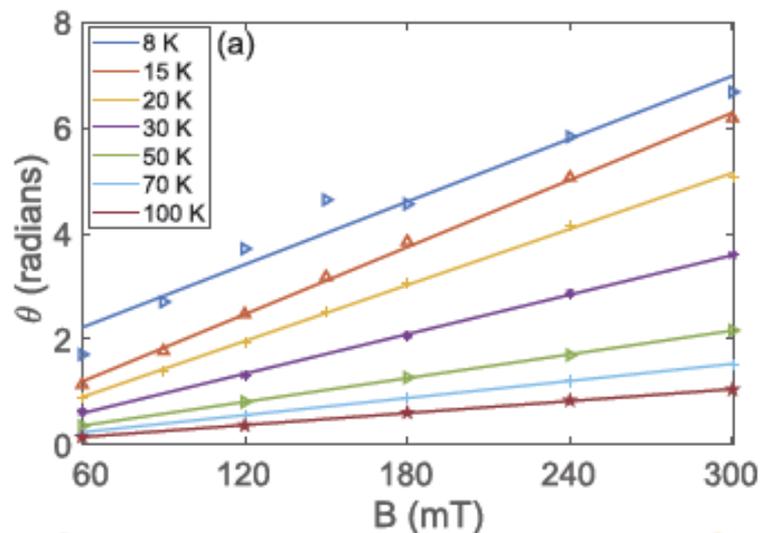
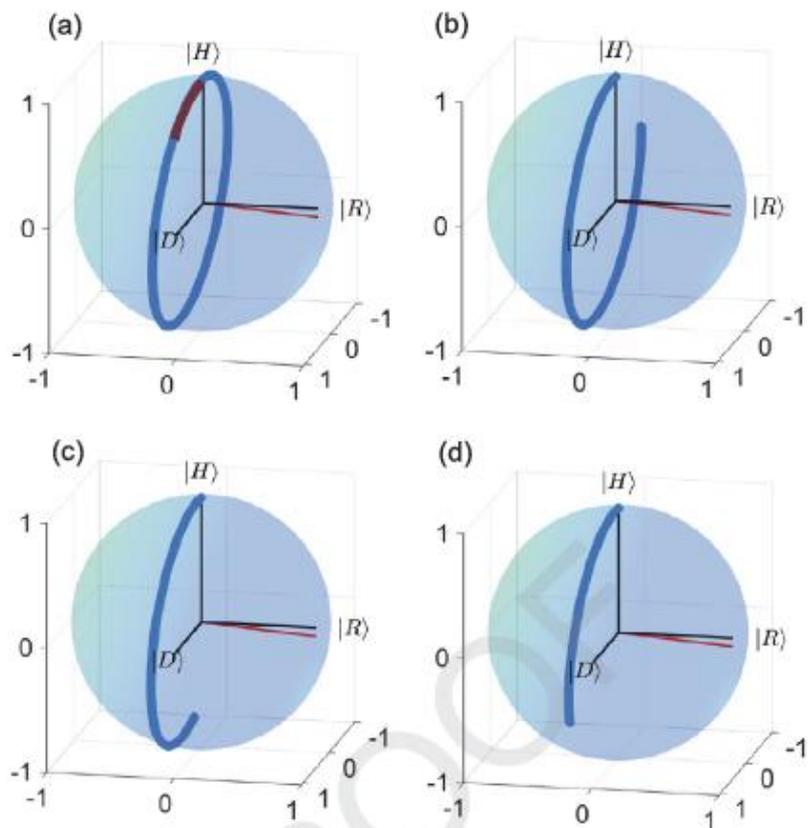
State	Predicted density matrix	Measured density matrix	Fidelity
$ HH\rangle$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0.93 & -0.11 & 0.03 & -0.03 \\ -0.11 & 0.06 & 0.00 & 0.01 \\ 0.03 & 0.00 & 0.00 & 0.00 \\ -0.03 & 0.01 & 0.00 & 0.01 \end{pmatrix}$	0.93
$ VV\rangle$	$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0.02 & -0.01 & -0.01 & 0.07 \\ -0.01 & 0.02 & 0.01 & -0.10 \\ -0.01 & 0.01 & 0.05 & 0.05 \\ 0.07 & -0.10 & 0.05 & 0.91 \end{pmatrix}$	0.91
$\frac{ HH\rangle+ VV\rangle}{2}$	$\begin{pmatrix} 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.49 & 0.05 & 0.01 & 0.30 \\ 0.05 & 0.05 & 0.02 & 0.01 \\ 0.01 & 0.02 & 0.05 & -0.01 \\ 0.30 & 0.01 & -0.01 & 0.41 \end{pmatrix}$	0.75

Quantum process tomography of a magneto-optic transformation

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$$\chi^F = \begin{pmatrix} 0.71 & -0.024 i & 0.45 i & 0.051 i \\ 0.024 i & 0.00 & -0.015 & 0.00 \\ -0.45 i & -0.015 & 0.29 & 0.032 \\ -0.051 i & 0.00 & 0.032 & 0.00 \end{pmatrix}$$



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