Of balls, bladders, and balloons: The time required to deflate an elastic sphere

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Experience teaches us that a large balloon takes longer to deflate than a small one of the same kind. But what is the quantitative relation between the deflation time τ and the radius R of a balloon? A simple analysis, depending only upon elementary physics, shows that $\tau \sim R^{7/2}$—a prediction that is surprisingly easy to illustrate with a party balloon, a tape measure, and a smart-phone app.
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I. INTRODUCTION

A balloon is comprised of elastic material that, if not stretched too far, returns, when relaxed, to its initial state. If the balloon is spherical, it can, in one's imagination, be divided into two hemispheres with a bisecting plane of area πR^2.[1] The net downward force exerted by the air (inside and outside the balloon) on the lower hemisphere of the balloon is $(P - P_{atm})\pi R^2$, where P is the pressure of the gas inside the balloon and P_{atm} is atmospheric pressure. This force is balanced by the upward force $\gamma 2\pi R$ exerted by the upper wall of the balloon on the lower wall. Here, γ is the surface tension supplied by the elastic material. The result is that $(P - P_{atm})\pi R^2 = \gamma 2\pi R$ or, equivalently

$$P = P_{atm} + \frac{2\gamma}{R}. \tag{1}$$

In ours and other applications, the surface tension γ is constant in the regime in which Eq. (1) obtains.

The "Young–Laplace equation"[2] (1) works well as long as the balloon radius R is larger than about one and a half times the radius of the balloon R_o when filled without excess pressure and smaller than approximately three or four times this radius.[1,3,4] In practice, we found that, as a rule of thumb, one strenuous breath inflated a party balloon to the regime in which Eq. (1) obtains. And, as long as further inflation becomes less difficult, the balloon remains spherical and the Young–Laplace law applies.

The Young–Laplace equation has numerous applications in medical physics, in the context of which it is called "The Law of Laplace," including for bladders and other animal organs.[5] While standard physics texts ignore this equation and its physics, medical physics texts[7] and pedagogically oriented monographs do not.[4,6]

For a party balloon, the difference between the pressure inside P and the pressure outside P_{atm} is typically small. After all, the pressure that can be exerted by human lungs is no more than, and typically much less than, approximately 10% of atmospheric pressure.[8–10] This means that, in this case, the inequality

$$P \approx P_{atm} \gg \frac{2\gamma}{R}, \tag{2}$$

is obtained.

II. DEFLATION TIME

How much time is required to deflate an elastic sphere through an opening of fixed size? Very little, according to the dramatic experience of Charlton Athletic, an English soccer club, that saw two balls burst in two consecutive FA Cup Final appearances (English soccer's equivalent of the Super Bowl).[11] Suppose the air leaves the sphere in streamline fashion, so that Bernoulli's law is observed and, consequently, the quantity $P + \rho v^2/2$ is constant along a streamline. Then the speed v of the air leaving the sphere at its opening is related to the pressures inside P and outside P_{atm} by

$$P = P_{atm} + \frac{\rho_{atm} v^2}{2}, \tag{3}$$

where ρ_{atm} is the mass density of the air at local atmospheric pressure P_{atm}. Note that we assume the air speed inside the sphere vanishes except near its opening. The rate at which mass m leaves the sphere is, according to the equation of continuity,

$$\begin{aligned} \frac{dm}{dt} &= -A\rho_{atm} v, \\ &= -A\sqrt{\frac{4\rho_{atm}\gamma}{R}}, \end{aligned} \tag{4}$$

where A is the cross-sectional area of the sphere's opening, and we have used Eqs. (1) and (3). If we had an independent relation between the total mass m of the air inside the sphere and its radius R, we could integrate Eq. (4) to obtain the time required for the sphere to deflate.

We find this relation in the following way. The air in the elastic sphere observes the ideal gas law $PV = Nk_BT$, which we express as

$$P = \frac{m}{m_o}\frac{k_BT}{V}, \tag{5}$$

where m is the mass of the air inside the sphere and m_o is the average mass of one molecule of air. Here, k_B is Boltzmann's constant and T is the air temperature. The volume V of the gas inside the sphere is related to its radius R by

$$V = \frac{4}{3}\pi R^3. \tag{6}$$

Using Eq. (5) to eliminate P from Eq. (1) produces

$$m = \left(\frac{m_o V}{k_B T}\right)\left[P_{atm} + \frac{2\gamma}{R}\right]. \tag{7}$$

Using Eq. (6) to eliminate V in Eq. (7), we find that

$$\begin{aligned} m &= \left(\frac{m_o}{k_B T}\right)\left(\frac{4\pi R^3}{3}\right)\left[P_{atm} + \frac{2\gamma}{R}\right], \\ &= \left(\frac{4\pi}{3}\right)\left(\frac{m_0 P_{atm}}{k_B T}\right)\left[R^3 + \frac{2\gamma}{P_{atm}}R^2\right], \\ &= \left(\frac{4\pi}{3}\right)\rho_{atm}\left[R^3 + \frac{2\gamma}{P_{atm}}R^2\right], \end{aligned} \tag{8}$$

where $\rho_{atm} = m_o P_{atm}/k_B T$ provides the desired relation between m and R.

Together the equation of continuity (4) and the relation (8) produce

$$-A\sqrt{\frac{4\rho_{atm}\gamma}{R}} = \left(\frac{4\pi}{3}\right)\rho_{atm}\left[3R^2 + \frac{4\gamma}{P_{atm}}R\right]\frac{dR}{dt}, \tag{9}$$

from which follows:

$$-A\sqrt{\frac{\gamma}{\rho_{atm}}}dt = 2\pi\left[R^{5/2} + \frac{4\gamma}{3P_{atm}}R^{3/2}\right]dR. \tag{10}$$

Integrating Eq. (10), from $R = R$ when $t = 0$ to $R = R_o$ when $t = \tau$, we find that the time τ required for an elastic sphere of radius R to deflate to its unpressurized radius R_o is

$$\tau = \frac{4\pi}{7A}\sqrt{\frac{\rho_{atm}}{\gamma}}\left[\left(R^{7/2} - R_o^{7/2}\right) + \frac{28}{15}\frac{\gamma}{P_{atm}}\left(R^{5/2} - R_o^{5/2}\right)\right]. \tag{11}$$

According to this result, the larger the sphere radius R, and, therefore, the larger the mass of the air it contains, the longer the deflate time τ. Also, the smaller the area A of its opening, the larger τ. Larger surface tension γ typically means shorter deflation times τ because, as we shall see, the first term in the square brackets on the right hand side of Eq. (11) dominates the second term. As an example of the dependence of the deflate time on atmospheric mass density ρ_{atm}, consider that otherwise identical elastic spheres, identically inflated, will deflate about 10% more quickly in Santa Fe, NM (at an of elevation 2133 m) than in Washington, DC (elevation 0 m) assuming that an isothermal atmosphere regulates atmospheric pressure and mass density.

III. EXPERIMENTS

Equation (11) can be empirically demonstrated with a party balloon, a flexible tape measure (to measure the balloon circumference), and a smart-phone application called Voice Memos (to measure the duration of the sound of air rushing from the balloon). It is important that the inflated balloon remains spherical and that the cross-sectional area A of the balloon mouth is constant. Accordingly, we inserted a snugly fitting segment of garden tubing into the balloon mouth and did not inflate the balloon beyond the limited regime in which it remained spherical.

Figure 1 shows a plot of the deflation time τ (in seconds) versus the radius R (in meters) for a common "12-in." party balloon. Filled circles are measurements, and the curve is from Eq. (11). The cross-sectional area of the balloon mouth is $A = 1.79 \times 10^{-5}\,\mathrm{m}^2$, and its unpressurized radius is $R_o = 0.024\,\mathrm{m}$. The atmospheric pressure in Santa Fe, New Mexico, USA (where the experiments were done) $P_{atm} = 78.1 \times 10^3$ Pa and the mass density of the atmosphere $\rho_{atm} = (78.1/101)1.225\,\mathrm{kg/m^3}$ are standard values. Finally, we chose a surface tension parameter $\gamma = 500\,\mathrm{kg/s^2}$ to fit the data.

Figure 2 shows a plot of the deflation time τ (in seconds) versus the radius R (in meters) for a super-sized "36-in." party balloon. The filled squares are measurements and the curve is from Eq. (11). The cross-sectional area of the balloon mouth is $A = 4.97 \times 10^{-5}\,\mathrm{m}^2$, and the radius of the unpressurized balloon is $R_o = 0.073\,\mathrm{m}$. The atmospheric pressure P_{atm} and the mass density of the atmosphere ρ_{atm} are the same as for the smaller balloon. While there is no reason the balloons should have exactly the same surface tension, $500\,\mathrm{kg/s^2}$, we found that this assumption works well.

In both cases, the pressures, P and P_{atm}, surface tension γ, and balloon radius R observe the inequality $P_{atm} \approx P \gg 2\gamma/R$. This suggests that the second term in the square brackets of Eq. (11) is small compared to the first term. Note, also, that the unpressurized radii of the small and of the large balloons are, respectively, 2.4 cm and 7.3 cm. Consequently, the ratio $(R_o/R)^{7/2}$ varies for the small balloon data from 8% to 0.4% and for the large balloon data from 22% to 1.8%. The data itself suggest that random variations associated with the difficulty of measuring R are at least as large as these more systematic variations. For these reasons, the power law

$$\tau = \left(\frac{4\pi}{7A}\right)\sqrt{\frac{\rho_{atm}}{\gamma}}R^{7/2} \tag{12}$$

derived from the largest term in Eq. (11) departs almost unnoticeably from the curves plotted in Figs. 1 and 2.

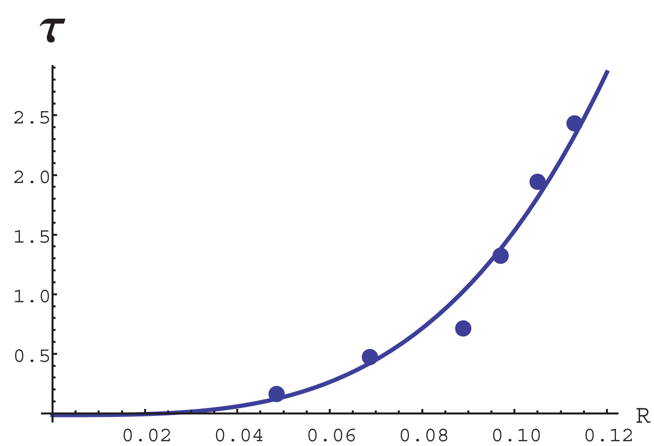

Fig. 1. Deflation time τ (in seconds) versus balloon radius R (in meters) of a common "12-in." party balloon. Filled circles: data. Solid line: Eq. (11) with $A = 1.79 \times 10^{-5}\,\mathrm{m}^2$, $P_a = 78.1 \times 10^3$ Pa, $\rho_{atm} = 0.947\,\mathrm{kg/m^3}$, $R_o = 0.024\,\mathrm{m}$, and $\gamma = 500\,\mathrm{kg/s^2}$.

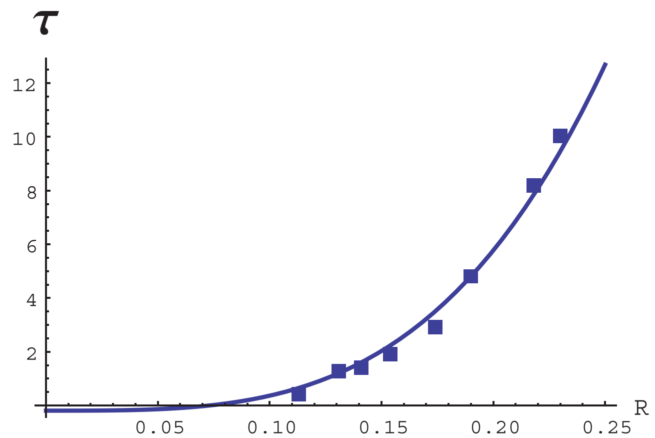

Fig. 2. Deflation time τ (in seconds) versus balloon radius R (in meters) of a common "36-in." party balloon. Filled circles: data. Solid line: Eq. (11) with $A = 4.97 \times 10^{-5}\,\mathrm{m}^2$, $P_a = 78.1 \times 10^3$ Pa, $\rho_{atm} = 0.947\,\mathrm{kg/m}^3$, $R_o = 0.024$ m, and $\gamma = 500\,\mathrm{kg/s}^2$.

The scaling $\tau \propto R^{7/2}$ is easily observed in the log-log plot displayed in Fig. 3. Here, the straight lines are the common logarithm (that is, to base 10) of the left hand side of Eq. (12) versus the common logarithm of R. The upper line is for the smaller balloon with mouth of area $A = 1.79 \times 10^{-5}\,\mathrm{m}^2$, and the lower line is for the larger balloon with mouth of area $A = 4.97 \times 10^{-5}\,\mathrm{m}^2$. The data points are the common logarithms of those plotted in Figs. 1 and 2. The slopes of the two lines are 7/2. Of course, the extent to which the individual data points depart from these lines, especially for the larger balloon, could be a reflection of the fact that R_o/R is not always ignorably small compared to one.

In principle, a slope and y-intercept of the straight lines in Fig. 3 could be found algorithmically rather than, as we have, by first assuming the slope to be 7/2 and second by choosing γ "by eye." However, we have found that with so few data points, the most divergent ones representing the smallest radii of each balloon have an undeserved influence on the algorithmic result. If desired, more data could help remedy this statistical problem.

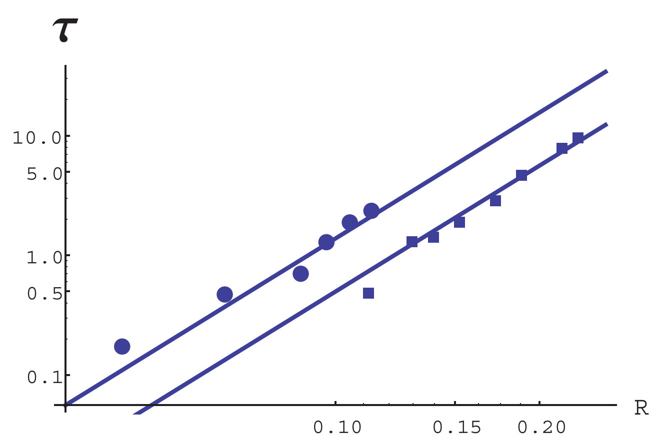

Fig. 3. Log-Log plot of the deflation time τ (in seconds) versus the balloon radius R (in meters) for the data shown in Fig. 1 (filled circles) and Fig. 2 (filled squares). The lines are log-log plots of both sides of Eq. (11) with the upper line for the smaller balloon with mouth of area $A = 1.79 \times 10^{-5}\,\mathrm{m}^2$ and the lower line for the larger balloon with mouth of area $A = 4.97 \times 10^{-5}\,\mathrm{m}^2$.

IV. EXTENSIONS AND VARIATIONS

Those who want to reproduce these experiments either at home or in a physics laboratory should be aware of the possibilities and the challenges. For instance, a stopwatch could substitute for the cell phone application Voice Memos but, probably, with less precision. Also, digital pressure gauges, of a type common in physics teaching laboratories, would allow one to use the Young–Laplace law (1) to determine the surface tension parameter γ directly instead of inferring its value from the balloon data. An amusing variation on this measurement of γ is to insert a smartphone into a semi-transparent balloon and record the pressure from its barometer application, as it is being inflated or deflated.[10]

Be forewarned that the small party balloon, if inflated and deflated too often, can change its elastic constant γ, or, worse yet, develop weak spots that bulge non-uniformly. For best results, use a new balloon. The larger balloon requires a larger tube at its mouth, taken from a segment of garden hose, than that required by the small balloon for which a segment of drip-system, irrigation tubing suffices.

The balloons we used are relatively inexpensive. Higher quality and more expensive, inflatable, elastic spheres are also available but were not tested.[12] We know from experience that a single person can perform these experiments. But a team of two might produce even better data.

V. CONCLUSION

We have asked and answered the question, "How much time τ is required to deflate an elastic sphere with radius R through an opening of definite cross-sectional area A?" Our derivation of the answer $\tau = (4\pi/7A)\sqrt{\rho_{atm}/\gamma}\,R^{7/2}$, where ρ_{atm} is the mass density of the atmosphere and γ is the (assumed constant) surface tension of the sphere, makes use of Bernoulli's principle, the equation of continuity, and the neglected, but useful, Young–Laplace equation. This relation is remarkably easy to illustrate with a party balloon, a flexible tape measure, and a smart-phone application called Voice Memos and as such could be the basis of a laboratory or home exercise.

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[1]The argument in this paragraph is taken from W. A. Osborne, "The elasticity of rubber balloons and hollow viscera," Proc. R. Soc. London **81**(551), 485–499 (1909).

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Expansion Bomb

You can use this device only once! It is made of cast iron, fairly thin-walled. Before the demonstration, it is filled to the top with water and the plug is screwed in tightly. It is then placed *outdoors* in freezing weather, and after a time it breaks apart. The trick is that water, unlike most metals, expands as it freezes. This device has been awaiting its use at Union College for a long time. (Picture and text by Thomas B. Greenslade, Jr., Kenyon College)