Properties of rubber balloons: addtitional notes

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Balloons are essential components of celebrations. Balloons are used to decorate events, whether for celebrations or to recognise accomplishments. Interestingly, Michael Faraday invented balloons in 1824, not for decoration, but to conduct experiments on gas laws. Experiments with balloons can help us understand how rubber-like materials behave. The goal of this experiment is to use balloons to investigate and validate known rubber structural models using well-known balloon inflation and deflation mechanisms. Because balloons are similar to organs like the heart, lungs, bladder, and arteries, this seemingly simple exercise is beneficial. Balloon stents are also used to open clogged arteries.

1 Theoretical Background: Pressure inside the balloon

Anybody who has inflated a balloon is aware that, initially, more pressure is required to inflate the balloon and subsequently becomes easier. The purpose of this lab experiment is to:

1. Explain the theoretical model.
2. Obtain the pressure-time curve of the rubber balloon.

1.1 Boyle's Law

Boyle's law states that the pressure and volume are inversely proportional to each other; that is, that product PV is a conserved quantity. If this was true, the pressure in the balloon would decrease with expansion and increase with contraction. According to Boyle's law:

$$PV = nRT, \tag{1}$$

where P is the pressure, V is the volume, n is the number of molecules of gas, R is the gas constant and T is the temperature. Care must be taken when applying Equation 1. This holds true for a closed system in which there is no exchange of particles with the outside world. In our case, we change n; therefore, this inverse relationship does not have to hold. Therefore, Boyle's law cannot be used to explain the relationship between the pressure inside a balloon and its radius when inflating or deflating it.

1.2 Young-Laplace equation

A different theoretical framework must be applied to describe the relationship between the pressure inside the balloon and the balloon radius. One approach is to use Laplace's law for the droplets and bubbles. This framework applies only to spherical objects, which is a reasonable scenario in our experiment. The Young-Laplace equation is defined as [1]:

$$\Delta P = \frac{2\gamma}{R}, \tag{2}$$

where P is the pressure, γ is the wall tension, and R is the balloon radius. The details of this derivation are provided in Appendix A.

Wall tension is a function of R; therefore, Equation 2 alone is not sufficient to describe the effect of the radius on the pressure inside the balloon. We need another model.

1.2.1 Hyperelastic Materials: Mooney-Rivlin Model

The law of elasticity must be included to explain the rubber deformation. The first option is to use Hooke's law. Hooke's law describes the linear relationship for elastic materials as follows:

$$F = -kx, \tag{3}$$

where F denotes the force applied to the balloon, k denotes the spring constant, and x denotes the amount of extension. However, it is well known that this equation does not describe the behavior of rubberlike materials [2]. Rubber possesses a response that is clearly different from the normal behavior of common elastic materials, for which Hooke's law applies: the strain is proportional to the applied stress. If we pull a piece of rubber band apart, we find that there is a relatively flat plateau after an initial increase in stress, similar to Hooke. Here, the stress is reasonably constant. With further stretching, the stress increases steeply because the polymer chains constituting the rubber become fully stretched .

Rubber is a class of materials known as hyperelastic materials. Such materials possess the following properties:

1. A nonlinear stress-strain response, which means that the slope of the curve changes with instantaneous strain.

2. Large elastic deformation with no plastic deformation.

3. Fully incompressible response.

From literature, it is known that the Mooney–Rivlin model fits the nonlinear stress-strain response of rubber. This mathematical model defines the relationship between the air pressure inside a spherical rubber balloon and its radius as [3]:

$$P = K\left[\frac{R_0}{R} - \left(\frac{R_0}{R}\right)^7\right]\left[1 + 0.1\left(\frac{R}{R_0}\right)^2\right], \tag{4}$$

where R is the radius, R_0 is the initial radius, K is a constant dependent on material properties and P is the internal pressure of the balloon.

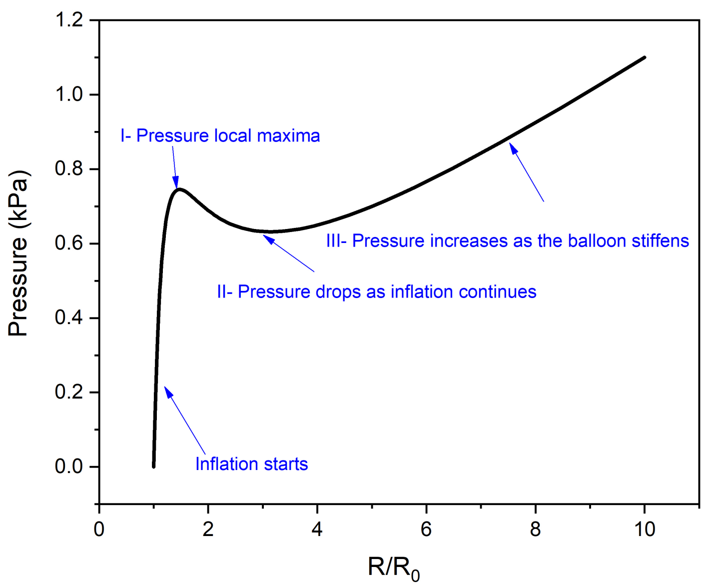

Figure 1: Theoretically predicted pressure variation with radius (see Equation 4).

Explanation: Figure 1 shows that there are three different pressure responses during the inflation phase. As we begin to inflate a balloon, there is (I) initially a sharp increase in the pressure, (II) then the pressure dips, and (III) as the balloon expands, we observe that the pressure increases again, eventually bursting.

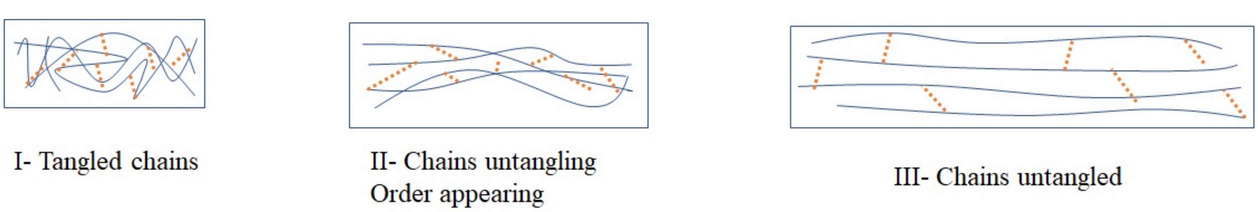

Figure 2: Untangling of polymer chains with the application of stress.

This behaviour can be explained by examining the material composition of the balloon. Balloons are made of rubber. Essentially, this elastic material is made up of flexible polymer chains that are all coiled up. A chemical bond exists within these chains that joins them together. Here is a plausible explanation to what is happening in the three regions, I through III.

Region I: Initially, when we begin to pump air, the internal pressure rises, the balloon expands, and these chains begin to straighten. The existing bonds stretch and exert restoring force. Initially, we need to apply more pressure to overcome this restoring force, which manifests as a sharp increase in the internal pressure during the initial seconds.

Region II: But, as these chains elongate further and get more organized, the overall effect is that the rubber balloon becomes stretchier and requires less pressure for expansion. This results in the dip that we observe.

Region III: Furthermore, as elongation continues, these chains straighten out fully, and the rubber material becomes stiff and harder to blow. This is because in this region, the links across the chains are being stretched. These are shown as connections in Figure 2. This is exhibited as a further increase in pressure as we continue to pump the air [4].

References

[1] D. S. Lemons, of balls, bladders, and balloons: The time required to deflate an elastic sphere, American Journal of Physics **89**, 80 (2021).

[2] D. R. Merritt and F. Weinhaus, The pressure curve for a rubber balloon, American Journal of Physics **46**, 976 (1978).

[3] Julien Vandermarlière, On the inflation of a rubber balloon, The Phys. Teach. **54**, 566 (2016).

[4] `https://www.mwmresearchgroup.org/the-science-of-balloons-part-1.html`.

A Derivation of the Young-Laplace Equation

Imagine a balloon consisting of two hemispheres with a bisection plane and a surface area of πR^2. Let the internal pressure be greater than the external one. There will be radially oriented outward forces in the upper half. Had all the arrows pointed in the same direction, the total force would have been:

$$F = \Delta PA = (P_{inside} - P_{outside})(2\pi R^2) \tag{A1}$$

However, forces are not pointing in the same direction, as all the horizontal components cancel each other, resulting in the total force in the upward direction being:

$$F = \Delta PA = (P_{inside} - P_{outside})(\pi R^2) \tag{A2}$$

This force is balanced by the downward force $\gamma 2\pi R$, which is exerted by the lower wall of the balloon, holding the upper half downward. γ denotes the surface tension of elastic balloon material. With these expressions,the Young-Laplace equation also known as Laplace bubble law is obtained as:

$$\Delta P = \frac{2\gamma}{R} \tag{A3}$$

where ΔP is the difference in pressure between the inside and outside of a spherical balloon. This derivation can also be performed by using the work method. The details can be found in Ref. [1]:

References

[1] https://thefactfactor.com/facts/pure_science/physics/laplaces-law/5349/.