Introduction to Quantum Information Science and Quantum Technologies

Assignment 3

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"Ferb, I know what we're gonna do today" - *Phineas*

Note

Submit yourID.ipynb or yourID.nb, containing the coding components and a hard copy of your solutions.

Question 1

Use Qiskit
Greenberger–Horne–Zeilinger states are quantum entanglement states containing three qubits. You may remember two of these states from the midterm:

$$|g\pm\rangle = \frac{1}{\sqrt{2}}(|000\rangle \pm |111\rangle)$$

(a) Construct quantum entangler circuit using *qiskit* which can produce the state $|g\pm\rangle$

(b) Evolve all possible inputs for the three-qubit systems to obtain the corresponding outputs using the *Statevector* library.

(c) From these results, construct the matrix form of this entangler operation. (Hint: Your results from the previous part give you the truth table corresponding to the operation.)

(d) Use this matrix or otherwise to derive the outer product form of this circuit.

Question 2

Use Qiskit
Construct the quantum teleportation circuit we developed in class.

We will use fidelity to quantify the accuracy of this quantum circuit. Which is defined as a measure of the "closeness" of two quantum states. Although its general description is for density operators, the formulation can be simplified to,

$$F(|\phi\rangle, |\psi\rangle) = |\langle\phi|\psi\rangle|^2$$

assuming that both density operators are pure.

(a) Show that the fidelity of the states, $|s\rangle = |0\rangle$, at completion of the scheme is unity.

(b) Introduce noise in the simulator and show reduced fidelity.

(c) Plot the fidelity relation against the noise introduced.

Question 3

Use Qiskit
For this question, you will implement the Deutsch-Joza Circuit for two functions, f_1 and f_2.

(a) Construct the oracles that implement these functions, $\hat{U}_{f_1}$ and $\hat{U}_{f_2}$, such that

$$f_1(x) = \begin{cases} 0 & x = \{00, 01\} \\ 1 & x = \{10, 11\} \end{cases}$$

$$f_2(x) = \begin{cases} 0 & x = \{00\} \\ 1 & x = \{01, 10, 11\} \end{cases}$$

(b) Make the DJ circuit for both oracles and show that you can correctly differentiate between the balanced and biased functions.

(c) Now, for the fun part, make another Deutsch-Joza circuit and add both $\hat{U}_{f_1}$ and $\hat{U}_{f_2}$ in place of the $\hat{U}_f$. Simulate this circuit and explain the results.

Question 4

Use Qiskit
The Grover search algorithm can be reduced to the operator $\hat{G} = \hat{V}_f\hat{O}_f$, where $\hat{V}_f$ and $\hat{O}_f$ are the diffuser and phase oracle respectively. Use the marked state,

$$|m\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |01\rangle)$$

(a) Run the Grover circuit and find the probability of getting the marked state after each iteration.

(b) Plot the probability of success against your result and determine the optimum number of iterations for this marked state.

Question 5

Use Qiskit where needed
In class, we developed the formulation behind the quantum Fourier algorithm and proposed the corresponding quantum circuit. You may consult your class notes.

(a) Find the Fourier matrix for qubit systems of size 4.

(b) Using this matrix or otherwise, find the quantum Fourier transform of the state

$$|\phi\rangle = \frac{1}{\sqrt{2}}(|0011\rangle + i\,|0010\rangle)$$

(c) Make the quantum Fourier circuit using *qiskit* for this and verify your answer for the last part.

Question 6

Use Qiskit where needed
Algorithms like Grover and phase estimation provide a computational advantage but are also integral to other algorithms. One such algorithm is the quantum counting algorithm.

(a) Make the Grover oracle $\hat{G}$ that marks and amplifies that state $|m\rangle = |11\rangle$.

(b) Construct the phase estimation algorithm using class notes but replace the unitary $\hat{U}$ with $\hat{G}$.

(c) Simulate your circuit and explain the results achieved.

Question 7

Use Qiskit where needed
Factor the number $N = 15$, using Shor's factoring algorithm, both the quantum and classical algorithms parts.

(a) Identify m, n the sizes of the respective registers.

(b) Construct the quantum circuit we discussed in class; you may use the inbuilt quantum Fourier gate.

(c) Plot the probability distributions for different values of a and example values of the order r.

Show the quantum states explicitly at each stage of the algorithm.