Quantum Image Processing - FRQI Image Representation

Gohar Raiz Haider 2022-12-0005
Wajeeha Rizwan 2022-12-0001
Syed Babar Ali School of Science and Engineering
Lahore University of Management Sciences

1 Abstract

Quantum Image Processing, as the name implies, is a method employed for processing images by using quantum information technology. It is a relatively new advancement in the field of Quantum Information Science and one that ensures time efficient management of simple operations used in classical image processing. The first and foremost step of this process is the encoding of classical images to quantum images which is done by a variety of different methods. In this paper the FRQI (Flexible Representation of Quantum Images) is explored in detail which encodes the images for representation on Quantum computers. The FRQI state consists information about the colours and their respective positions in the image. Once the FRQI state is achieved, the required Quantum Image Processing algorithm is applied to it which is needed for the specific purpose of carrying out the entire process. The FRQI is not only used for image representation but for various other related tasks of Quantum Image Processing. After the preparation of the FRQI state, its circuit implementation and simulation are carried out on Qiskit.

2 Introduction

Although a relatively new concept in the field of science and technology, quantum computing is being researched on and used extensively for finding solutions to problems that seem too complicated for the existing classical computers. Though the quantum computers are quite different and rather complex to use but the promise of exponentially increasing the speed and efficiency of problem solving has created an interest among researchers worldwide to dig more into the applications of this field. Quantum computers are basically even more advanced than the supercomputers. Even those problems that seem extremely complex for supercomputers, like the modelling of atoms in a compound, quantum computers can perform such tasks rather easily. Currently, quantum computers and quantum technology in general is being used in various applications like electric vehicles, solving of complex energy challenges, quest of solving space and cosmic mysteries, image processing and a variety of a lot of other applications [5].

One of the most interesting area where Quantum science and technology is being utilized is Quantum Image Processing. With a multitude of advances in technology, image processing has become an extensively researched upon area of technology. It is being employed in various different disciplines and areas of research. Facial recognition, automated vehicles, image Photoshop and numerous other techniques use image processing as their basis. Utilizing Quantum technology in image processing is what Quantum image processing is all about. As is evident until now, quantum image processing will be a lot more efficient in terms of speed and time than its classical counterpart. It will also prove to be extremely useful for simple day to day applications like simple face recognition on a mobile phone or criminal detection at a police station, but all of this might be possible in a fraction of time and with much less error than is now using classical techniques. Image compression, edge detection, image storage, image retrieval, binary image line detection are just some of the tasks achievable by quantum image processing [1, 4].

In order to carry out Quantum Image Processing, the image must first be transformed into its quantum counterpart known as the quantum image. This state can be achieved by a number of different processes like FRQI (Flexible Representation of Quantum Image), NEQR (Novel Enhanced Quantum Representation), QBIP (Quantum Boolean Image Processing) and a variety of others [6]. Here we discusses the FRQI technique in detail. The FRQI state represents the classical images, after a transformation, as quantum images on a quantum computer in a normalized state. This state carries the information about the colours in an image and their respective positions. It is quite an efficient method for the preparation of an image on which various different quantum image processing algorithms can be applied to achieve desired results. Not only does FRQI provide a representation to images but can also prove to be extremely useful for the exploration of other tasks performed by the quantum computers regarding image processing [1, 2]. Preparing this state requires a polynomial number of simple operations and gates [2, 4]. We work with a $2x2$ image i.e. 4 pixels image, where the color and its position is encoded in the FRQI state as shown below,

$$|I(\theta)\rangle = \frac{1}{2^n} \sum_{i=0}^{2^{2n}-1} (\cos(\theta_i)|0\rangle + \sin(\theta_i)|1\rangle) \otimes |i\rangle, \tag{1}$$

$$\theta_i \in [0, \frac{\pi}{2}], i = 0, 1, 2, ..., 2^{2n} - 1. \tag{2}$$

3 Mathematical Formulations

The FRQI state contains coded information in the form of colour and its related pixel position as shown below. FRQI state is prepared through a unitary transformation which has two steps. First applying the hadamard transform $\mathcal{H} = I \otimes H^{\otimes 2}$, where I is the $2D$ identity matrix and H is the hadamard gate, on $|0\rangle^{\otimes 3}$, producing the state $|H\rangle$,

$$(I \otimes H^{\otimes 2})|0^{\otimes 3}\rangle = \frac{1}{2}|0\rangle \otimes \sum_{i=0}^{3} |i\rangle = |H\rangle. \tag{3}$$

State $|0\rangle$ is initialised on all three qubits and hadamard gate is applied on the first two, creating superposition. The third qubit is our ancillary qubit. In the second step, controlled rotations are applied on the $|H\rangle$ state as defined by,

$$R_i = (I \otimes \sum_{j=0, j\neq i}^{3} |j\rangle\langle j|) + R_y(2\theta_i) \otimes |i\rangle\langle i|. \tag{4}$$

where

$$R_y(2\theta_i) = \begin{pmatrix} \cos(\theta_i) & -\sin(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i) \end{pmatrix}$$

The controlled rotations are applied in succession correspondimg to the number of pixels, which in our case is 4. This corresponds to a unitary operation $\mathcal{R}$ defined as,

$$\mathcal{R}|H\rangle = \prod_{i=0}^{3} R_i|H\rangle. \tag{5}$$

Equation '(1)' is our state obtained after applying the hadamard transform. Now, the controlled rotation operators are applied in succession as follows.

$$\begin{aligned} R_0|H\rangle &= \left(I \otimes \sum_{i=0, i\neq 0}^{3} |i\rangle\langle i|) + R_y(2\theta_0) \otimes |0\rangle\langle 0|\right) (\frac{1}{2}|0\rangle \otimes \sum_{i=0}^{3} |i\rangle) \\ &= \frac{1}{2}\left[|0\rangle \otimes \sum_{i=0, i\neq 0}^{3} |i\rangle\langle i| + (\cos(\theta_0)|0\rangle + sin(\theta_0)|1\rangle) \otimes |0\rangle\right]. \end{aligned} \tag{6}$$

$$\begin{aligned} R_1(R_0|H\rangle) = \left(I \otimes \sum_{i=0, i\neq 1}^{3} |i\rangle\langle i|) + R_y(2\theta_1) \otimes |1\rangle\langle 1|\right) \frac{1}{2}\Bigg(|0\rangle \otimes \sum_{i=0, i\neq 0}^{3} |i\rangle\langle i| \\ + (\cos(\theta_0)|0\rangle + sin(\theta_0)|1\rangle) \otimes |0\rangle]\Bigg) \end{aligned}$$

$$= \frac{1}{2}\left[|0\rangle \otimes \sum_{i=0, i\neq 0,1}^{3} |i\rangle\langle i| + (\cos(\theta_0)|0\rangle \sin(\theta_0)|1\rangle) \otimes |0\rangle + (\cos(\theta_1)|0\rangle + sin(\theta_1)|1\rangle) \otimes |1\rangle\right]. \tag{7}$$

$$\begin{aligned} R_2(R_1R_0|H\rangle) = \left(I \otimes \sum_{i=0, i\neq 2}^{3} |i\rangle\langle i|) + R_y(2\theta_2) \otimes |2\rangle\langle 2|\right) \frac{1}{2}\Bigg(|0\rangle \otimes \sum_{i=0, i\neq 0,1}^{3} |i\rangle\langle i| \\ + (\cos(\theta_0)|0\rangle + sin(\theta_0)|1\rangle) \otimes |0\rangle + (\cos(\theta_1)|0\rangle + sin(\theta_1)|1\rangle) \otimes |1\rangle\Bigg) \end{aligned}$$

$$= \frac{1}{2}\left(|0\rangle\otimes \sum_{i=0,i\neq 0,1,2}^{3} |i\rangle\langle i| + (\cos(\theta_0)|0\rangle + sin(\theta_0)|1\rangle)\otimes|0\rangle + (\cos(\theta_1)|0\rangle + sin(\theta_1)|1\rangle)\otimes|1\rangle \right.$$

$$\left. + (\cos(\theta_2)|0\rangle + sin(\theta_2)|1\rangle) \otimes |2\rangle \right). \quad (8)$$

$$R_3(R_2R_1R_0|0\rangle) = \left(I \otimes \sum_{i=0,i\neq 3}^{3} |i\rangle\langle i|) + R_y(2\theta_3) \otimes |3\rangle\langle 3| \right) \frac{1}{2}\left(|0\rangle \otimes \sum_{i=0,i\neq 0,1,2}^{3} |i\rangle\langle i| \right.$$

$$\left. + (\cos(\theta_0)|0\rangle + sin(\theta_0)|1\rangle)\otimes|0\rangle + (\cos(\theta_1)|0\rangle + sin(\theta_1)|1\rangle)\otimes|1\rangle \ + (cos(\theta_2)|0\rangle + sin(\theta_2)|1\rangle)\otimes|2\rangle \right)$$

$$= \frac{1}{2}\left(|0\rangle\otimes \sum_{i=0,i\neq 0,1,2,3}^{3} |i\rangle\langle i| + (\cos(\theta_0)|0\rangle + sin(\theta_0)|1\rangle\otimes|0\rangle + (\cos(\theta_1)|0\rangle + sin(\theta_1)|1\rangle\otimes|1\rangle \right.$$

$$\left. + (cos(\theta_2)|0\rangle + sin(\theta_2)|1\rangle) \otimes |2\rangle + (cos(\theta_3)|0\rangle + sin(\theta_3)|1\rangle) \otimes |3\rangle \right). \quad (9)$$

In above equation, the second term vanishes and $\sum_{i=0}^{3} |i\rangle$ are just numeric representation of two qubit states in Zeeman basis,

1. $|0\rangle = |00\rangle$
2. $|1\rangle = |01\rangle$
3. $|2\rangle = |10\rangle$
4. $|3\rangle = |11\rangle$.

Hence, the FRQI state achieved is,

$$\frac{1}{2}\left((\cos(\theta_0)|0\rangle + sin(\theta_0)|1\rangle) \otimes |00\rangle + (\cos(\theta_1)|0\rangle + sin(\theta_1)|1\rangle) \otimes |01\rangle \right.$$

$$\left. + (\cos(\theta_2)|0\rangle + sin(\theta_2)|1\rangle) \otimes |10\rangle + (\cos(\theta_3)|0\rangle + sin(\theta_3)|1\rangle) \otimes |11\rangle \right). \quad (10)$$

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