Quantum Tic Tac Toe

Course Project for PHY 318

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Abstract

In this project, we propose a game of Quantum Tic Tac Toe: a classical game of Tic Tac Toe, but with a quantum twist. With the implementation of quantum moves of "Collapse" and "Entanglement" in an accompanying, interactive quantum circuit, players can use the power of quantum mechanics to throw off an opponent or win the entire game. Such a quantum game aims to show the player (who may have a penchant for quantum physics) to see the inherently probabilistic nature of quantum mechanics and its other properties in real-time - and in a fun, interactive way.

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1 Introduction

Quantum mechanics is a complex field; concepts like entanglement, superposition and the like can all get quite abstract even for experts. Considering how popular this field has become, it is imperative that the fundamentals of quantum mechanics be made more palatable to the avidly interested student - and what better way to do so than through a quantum mechanical game. For this project, I have chosen to illustrate and explain a game of 3×3 tiled quantum tic tac toe, through its implementation on a quantum circuit. Through this, the goal is to show all the important fundamental aspects of [3]:

1. Superposition: the ability of a quantum state to be in two states at once
2. Entanglement: the phenomenon by which the state of one component can be influenced by the state of another component, through some kind of correlation
3. Collapse: the process through which a quantum state reduces - or "collapses" - to a classical state.

2 Mathematical Formulation

In this section, we shall explore the mathematics behind our quantum version of tic tac toe, which will accompany the main component of the game: the quantum circuit, with which both players are supposed to interact with. However, considering that the rules of the game have been kept deliberately simple, the aim in this portion of the paper is to provide the players with a kind of sketch as to how states within the tile are evolving as a game proceeds. Therefore, instead of keeping the players completely out of the loop about all the quantum mechanics hidden behind the game-board, instead the players are encouraged to explore the consequences of these quantum gates; through this, they may even be able to create ever-changing strategies following every step in order to win a game. [2]

First, we will present the gates that will be used as part of the rules of the game. Then, the initial configuration of the game-board will be presented, wherein each tile contains a superposition state of **X** and **O**. Finally, to show the gates inter-playing with the superposition states within the tiles - in accordance with the legal moves - a step-by-step game-play will be shown, wherein two versions of the game-board will be shown: one will be a "classical" game-board, which will show the positions of **X**'s and **O**'s after each move, and another will be a "quantum" game-board, which contains all the information about the gates used and states existing in each tile.

2.1 Gates Used

2.1.1 Hadamard Gate

This gate is a quantum logic gate that acts on a single qubit. Its sole responsibility is to create equal superposition of $|0\rangle$ and $|1\rangle$, the final form of which depends on what kind of qubit is it acting on:

$$U_H |0\rangle = \frac{1}{\sqrt{2}} \left(|0\rangle + |1\rangle\right),$$
$$U_H |1\rangle = \frac{1}{\sqrt{2}} \left(|0\rangle - |1\rangle\right).$$

Its matrix representation is as follows [1]:

$$U_H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{2.1}$$

For the purpose of our quantum game of tic tac toe, the Hadamard gate is used to create equal superposition of $|0\rangle$ and $|1\rangle$ in all the tiles at the beginning of the game.

2.1.2 CNOT Gate

This is a quantum logic gate that acts on two qubits instead of one. One qubit acts as the control, while the other as a target; therefore, these qubits are also called control and target qubits respectively. Its purpose is to flip the state of the target qubit when the control qubit is precisely $|1\rangle$. Its matrix form can be written as:

$$U_{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{2.2}$$

For the purpose of our game, this gate will be used to create entanglement with two states, such that they form two bell states (explained in more detail ahead).

2.2 Initial Configuration of the Board

We will start the way any tic tac toe game begins: by considering a 3×3 tiled game-board - except, in this quantum counterpart, each tile is not necessarily empty. Instead, each tile contains an equal superposition state of $|0\rangle$ and $|1\rangle$:

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|0\rangle + |1\rangle \right).$$

$\|\psi\rangle$	$\|\psi\rangle$	$\|\psi\rangle$
$\|\psi\rangle$	$\|\psi\rangle$	$\|\psi\rangle$
$\|\psi\rangle$	$\|\psi\rangle$	$\|\psi\rangle$

2.3 Rules

The end goal is the same as classical tic tac toe: the first one to get a complete row, column or diagonal filled with uninterrupted **X**'s or **O**'s wins. How to get there, though, requires a little more than just the simple rules of the classical game.

(a) Each player can do one of two legal plays: (1) collapse, and (2) entanglement.

(b) For the "Collapse" move, the player will have to chose one of the tiles, and then it will either get assigned $|0\rangle$ or $|1\rangle$, and each has a 50% probability. (And yes, both players will be well aware of that risk beforehand!) This move can happen legally in one of two types of tiles: (1) one that has not been occupied yet, and (2) a tile locked in a control terminal of a $CNOT$ gate.

(c) For the "Entanglement" move, the player will apply the $CNOT$ gate across two tiles. However, to make the game simpler, the target terminal must be placed at an already "collapsed" tile for it to be a legal move. That way, there are only two types of entangled bell states that are possible:

$$|B_1\rangle = U_{\text{CNOT}}\left(|\psi\rangle\,|0\rangle\right) = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right),$$
$$|B_2\rangle = U_{\text{CNOT}}\left(|\psi\rangle\,|1\rangle\right) = \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right).$$

This inclusion of "entanglement" allows a player to be able to present an obstacle for the opponent: instead of allowing them to maintain their tile, the other player can entangle their chosen tile with their opponent's, so as to render their occupied tile "smeared". The best way to show the efficacy of this move is by showing it in a real game.

2.4 Step-by-step Game Play

As mentioned in the beginning of this section, the following step-by-step game play is supposed to show the reader how our newly modified "quantum" moves can be applied by players competing to win. The right side contains the "classical" board - i.e. the board that the players will be able to see their results on. On the left hand side, there is the "quantum" board, which represents all the gates used and quantum states evolved as a result. Note that C in each tile represents a "Collapse" move placed there, and E represents an "Entanglement" move; for the sake of maintaining a history of all moves made in a complete game, all such C and E notations are retained in each tile. Gates that are colored black are operational, and when they turn grey, that is when they become obsolete or ineffective. In each move, we shall show each player's strategy as we go along; more precisely, we will show how knowledge of quantum states can motivate players to strategise their next move, so as to secure a win.

1. *Move 1 by Player* **X***: "Collapse" on Tile 5.*

1.	2.	3.
4.	5. **X**	6.
7.	8.	9.

1.	2.	3.
4.	5. $\|1\rangle$ C	6.
7.	8.	9.

2. *Move 2 by Player* **O***: "Collapse" on Tile 3.*

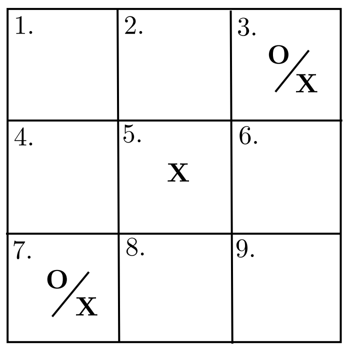
1.	2.	3. **O**
4.	5. **X**	6.
7.	8.	9.

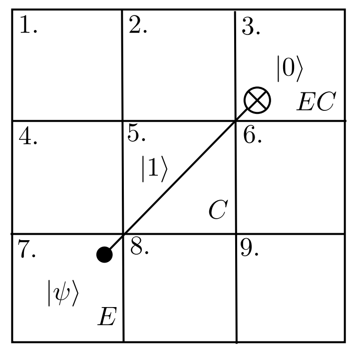
1.	2.	3. $\|0\rangle$ C
4.	5. $\|1\rangle$ C	6.
7.	8.	9.

3. *Move 3 by Player* **X***: "Entanglement" of Tile 7 (Control) with Tile 3 (Target).* At this stage, Player **X** wishes to get the upper-hand by securing the corners, while simultaneously attempt to topple Player **O**'s hold in Tile 3. Thus, he chooses the "Entanglement" move, which smears the information of **O** in Tile 3. The resulting state one gets is:

$$|\psi_{7\to 3}\rangle = U_{CNOT}\,|\psi\rangle\,|0\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right).$$

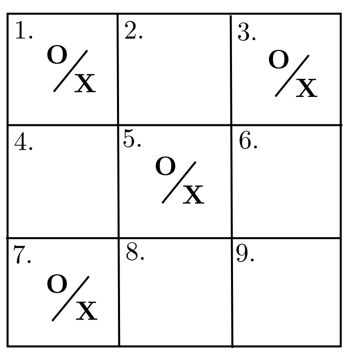
Notationally, $|\psi_{7\to 3}\rangle$ represents the entangled bell state wherein the qubit in Tile 7 is the control qubit and the qubit in Tile 3 is the target qubit. This new entangled state exists in both Tiles 7 and 3; what exact state resides in each tile is decided when a "Collapse" move is made in Tile 7, in which case there is a 50% probability of both tiles getting $|0\rangle$ or $|1\rangle$. Until then, both are in an entangled superposition of sorts, as shown in the figure below.

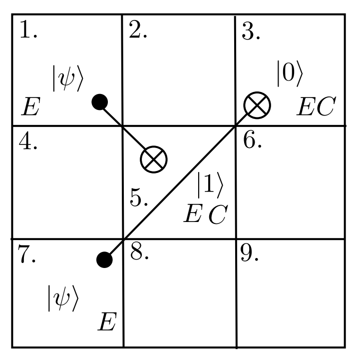




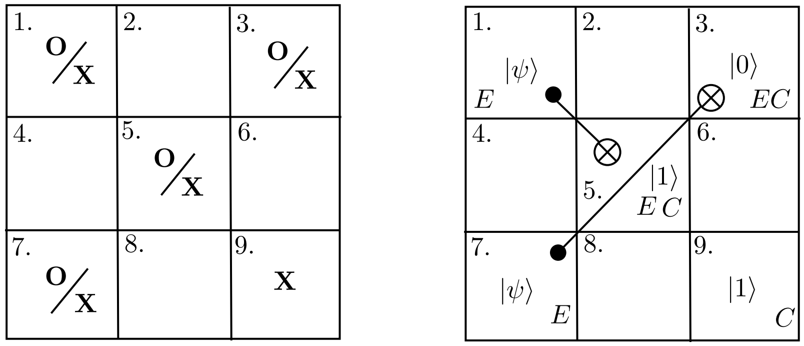
4. *Move 4 by Player* **O***: "Entanglement" of Tile 1 (Control) with Tile 5 (Target).* Now, Player **O** notices **X**'s trick, and so goes for a similar trick to loosen **X**'s grasp in the center tile; he entangles Tiles 1 and 5, such that the qubit in Tile 1 is the control qubit, and Tile 5 is the target qubit. This new entangled state is as follows:

$$|\psi_{1\to 5}\rangle = U_{CNOT}\,|\psi\rangle\,|1\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right).$$

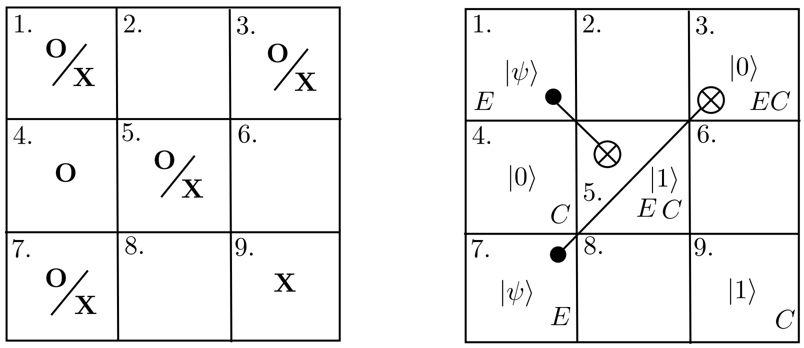




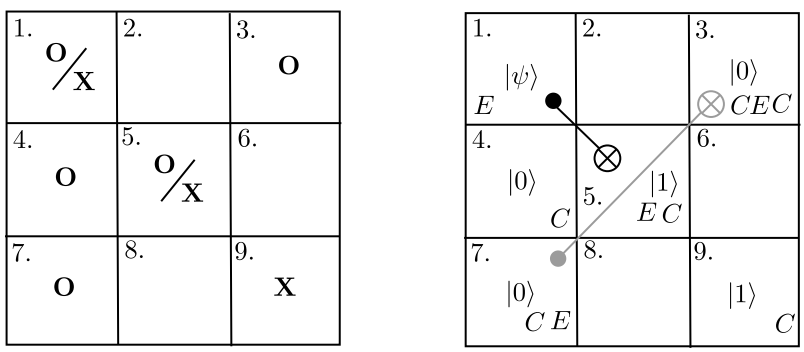
5. *Move 5 by Player* **X***: "Collapse" at Tile 9.*



6. *Move 6 by Player* **O***: "Collapse" at Tile 4.*



7. *Move 7 by Player* **X***: "Collapse" at Tile 7.* In an attempt to finally secure the corners, Player **X** takes the risk and collapses $|\psi_{7\to3}\rangle$. But to his dismay, the states chosen are $|0\rangle$ on both Tiles 7 and 3. The entanglement placed before is now rendered obsolete.



8. *Move 8 by Player* **O***: "Collapse" at Tile 1.* Player **O** now goes for the kill and chooses the "Collapse" move in Tile 1, for which there is a 50% probability of either of the following:

   (a) Tile 1: $|0\rangle$ and Tile 5: $|1\rangle$. In this scenario, Player **O** wins, as seen in Figure 1.

   (b) Tile 1: $|1\rangle$ and Tile 5: $|0\rangle$. In this scenario, Player **O** wins again, as shown in Figure 2.

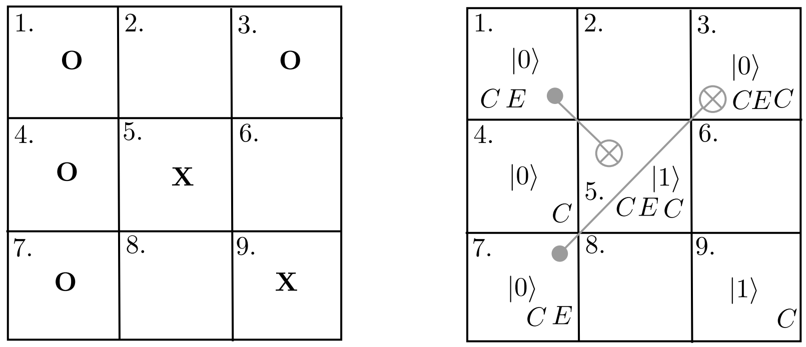


Figure 1: Tile 1 with $|0\rangle$ and Tile 5 with $|1\rangle$

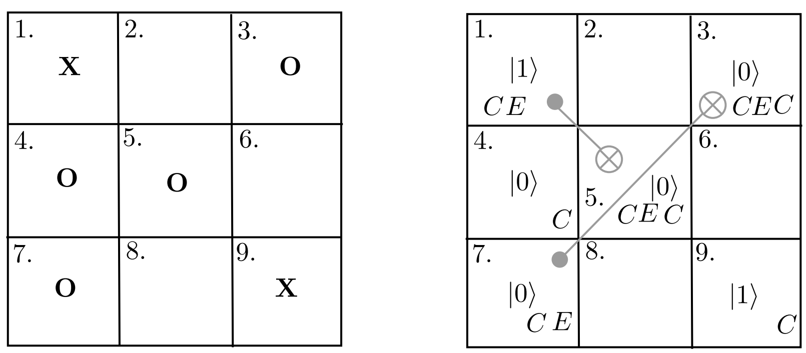


Figure 2: Tile 1 with $|1\rangle$ and Tile 5 with $|0\rangle$

3 Further Improvements

To add more layers of complexity, this game can be extended by adding more possible moves in the list of rules. So, instead of limiting a player's choice to pick "Collapse" or "Entanglement", they could also pick "Switch" and "Superposition":

1. "Switch" would involve using the *NOT* gate on a tile; however, to avoid any player getting an unfair advantage, the limit that could be placed is that the *NOT* gate can only be placed on entangled tiles (control or target); that way, one could very well switch the kind of entangled bell state residing in two qubits.

2. "Superposition" would involve using the Hadamard gate on a tile, as long as it contains a collapsed state; a player won't be allowed to use it on a tile that already contains a superposition state, otherwise it would only collapse back to $|0\rangle$ due to unitarity.

4 Bibliography

References

[1] M. A. Nielsen and I. L. Chuang, "*Quantum Computation and Quantum Information*", Cambridge University Press, (2010), pp. 500-501.

[2] M. Nagy and N. Nagy, "Quantum Tic-Tac-Toe: A Genuine Probabilistic Approach."

[3] W. Maurice, W. Tim, "Quantum Tic-Tac-Toe - learning the concepts of quantum mechanics in a playful way", *Computers and Education Open 4*, (2022).