Modeling Projectile Motion using a Trebuchet

Group 8
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1 Conceptual Objectives

In this experiment we:

1. Research the history of a trebuchet
2. Look at several models to understand all the essential features of a trebuchet
3. Sketch a model of the trebuchet and figure out which dimensions give the best results
4. Build and test out the trebuchet
5. Test the model with variable masses and variable throwing objects
6. Figure out how the mass of the counterweight and the throwing object affect the range of the projectile
7. Calculate uncertainties related to the experimental procedure
8. Plot the readings from the experiment in MATLAB
9. Propagate the uncertainties in MATLAB

2 Experimental Objectives

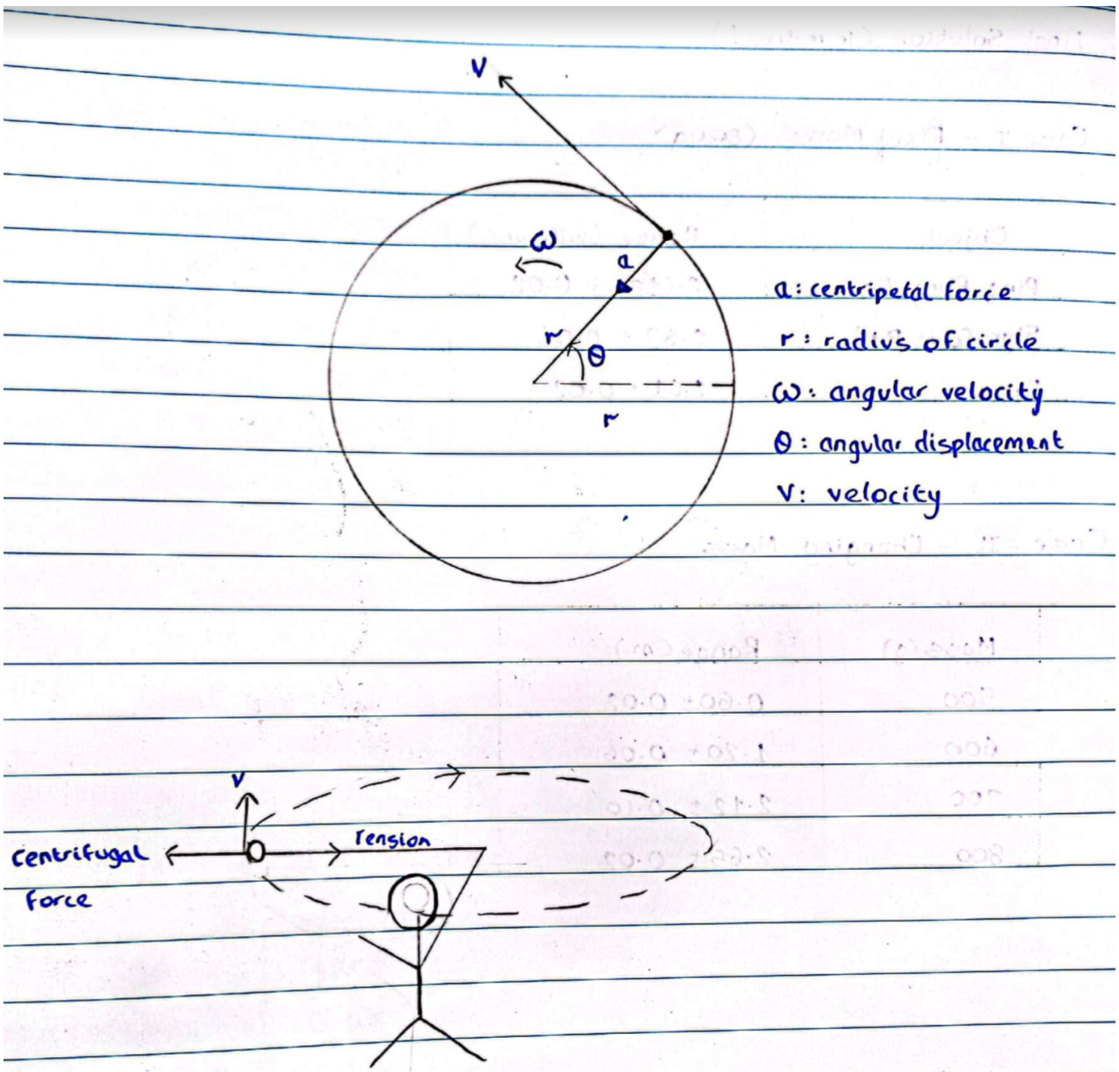
The first part of this experiment is researching and producing a bill of materials for building a model suitable for the lab. The second part consists of testing out the model with variable masses and recording readings for the range covered, while keeping the throwing object constant. The third part consists of testing different throwing objects with a constant mass and recording their range. Lastly, the uncertainties in the experiment are calculated and plots of range vs. mass of counterweight and range vs. mass of object are plotted with their uncertainties.

3 Theoretical Introduction

This experiment introduces us to the basic concepts of circular and projectile motion. We will learn how the dimensions of the throwing arm and ratio of the short arm to the long arm affects the range of the projectile. Furthermore, we will discuss what type of string and payload needed to effectively launch the projectile.

3.1 Circular Motion

Circular motion is described as a movement of an object while rotating along a circular path. Circular motion can be either uniform or non-uniform. During uniform circular motion, the angular rate of rotation and speed will be constant, while during non-uniform motion the rate of rotation keeps changing.



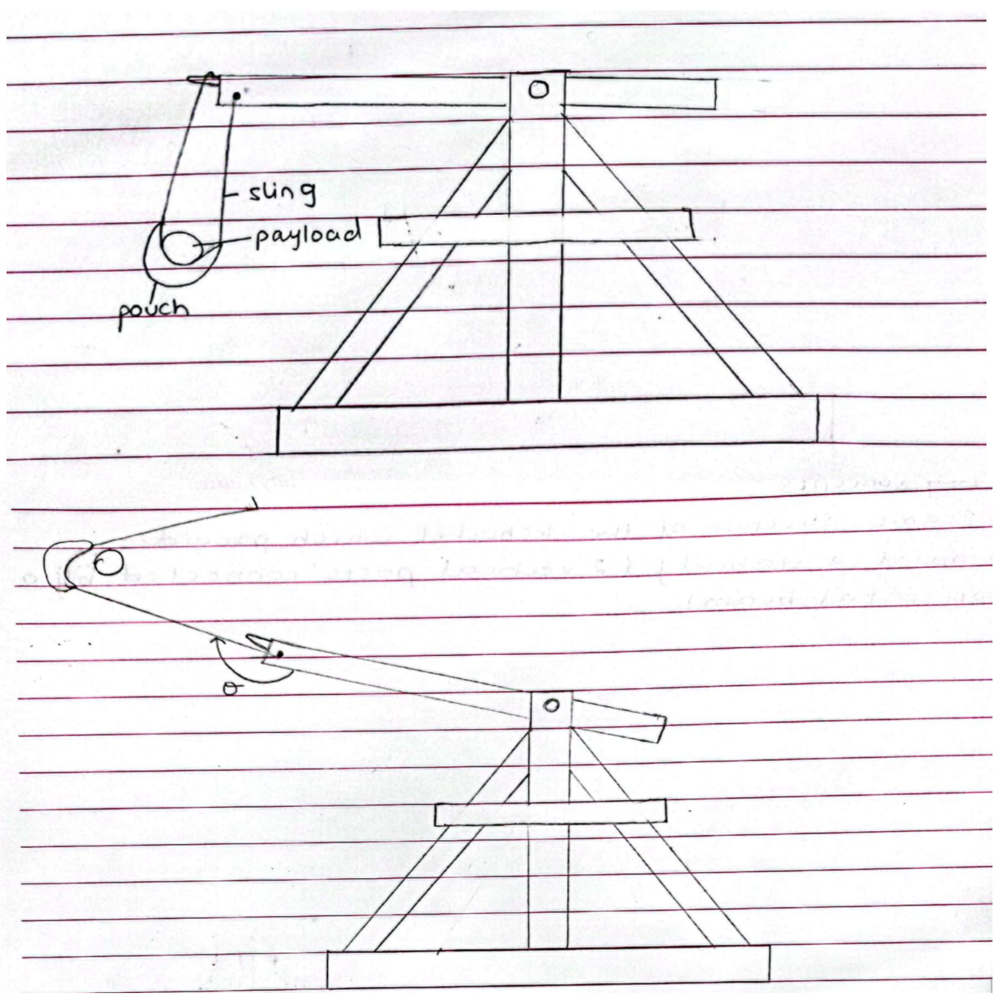
3.2 Concepts Used

The trebuchet relies heavily on potential energy stored in the counterweight. The counter weight is raised to a certain height before its launch and this imparts potential energy in the system. When the mass is accelerated, the potential energy is converted to kinetic energy increasing the counterweights power on impact.The counterweight is affected by gravity when the trebuchet is released. As the counterweight descends, potential energy is transformed into kinetic energy by the force of gravity (mg) acting on it.

Intuitively, you might think that the angle of release should be $\alpha = 45°$. Let's see if this holds true for our trebuchet model.

The pouch is attached to a string which is fixed to the long arm of the lever on one side and loosely attached to the end of the long arm. When the sling with the object is pulled down and released its speed is great enough to keep the projectile in the pouch. The loose end of the string then slips off the arm giving the sling a sudden jerk which will cause the object to be released from the pouch and follow the path of a projectile.

The loose end of the string could be attached directly to the long arm using a loose knot or it can be attached to a nail at the end of the long arm. Both ways can be tested to see which one gives the best projectile motion.

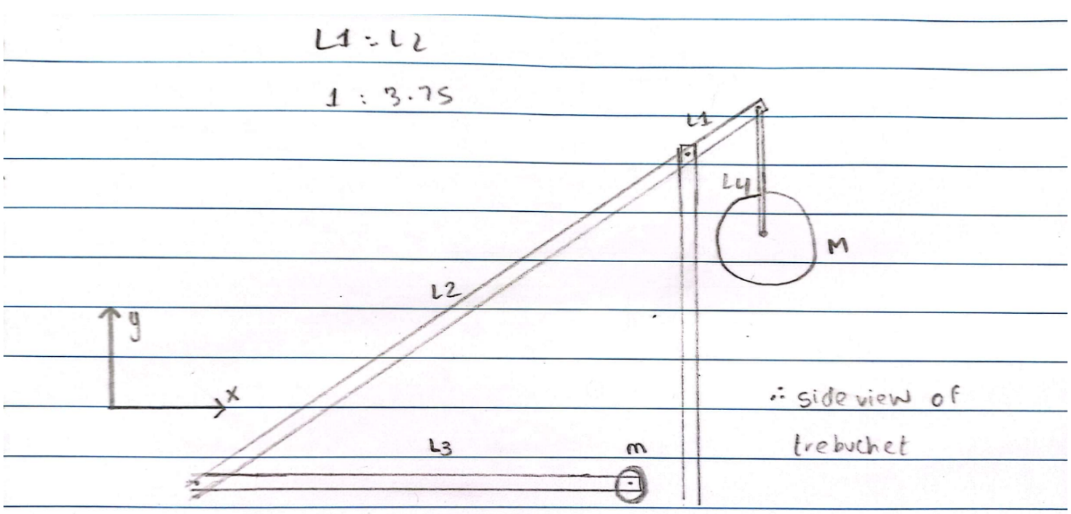


4 Introduction to the Apparatus

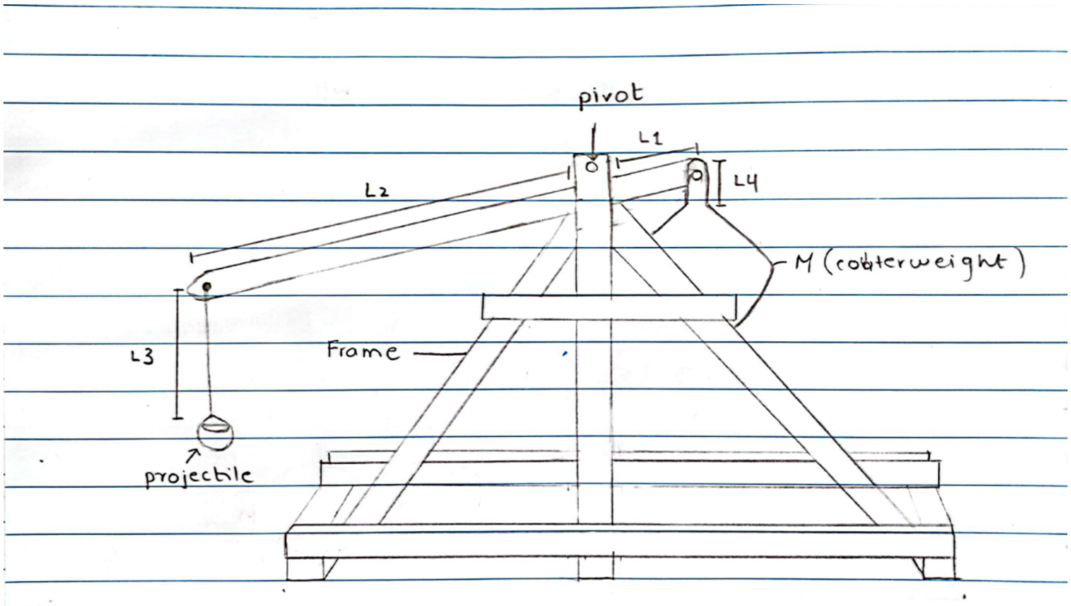
- **Frame**: the structure of the trebuchet which provides support and stability. It consists of two vertical posts connected by a horizontal beam.
- **Counterweight**: a heavy mass which is attached to the short end of the throwing arm. It provides the necessary energy for launching the projectile.
- **Throwing Arm**: a sort of lever which is attached to the frame. One end of the arm holds the sling and the other has the counterweight. As the counterweight falls it raises the throwing arm and upon release it launches the projectile.
- **Sling**: a pouch attached to the throwing arm. As the throwing arm swings the sling releases the projectile and propels it forwards. The sling is responsible for determining the trajectory and distance of the projectile.
- **Rollers**: rollers are put in place to reduce friction and help guide the throwing arm to the desired path.

Sketch of trebuchet

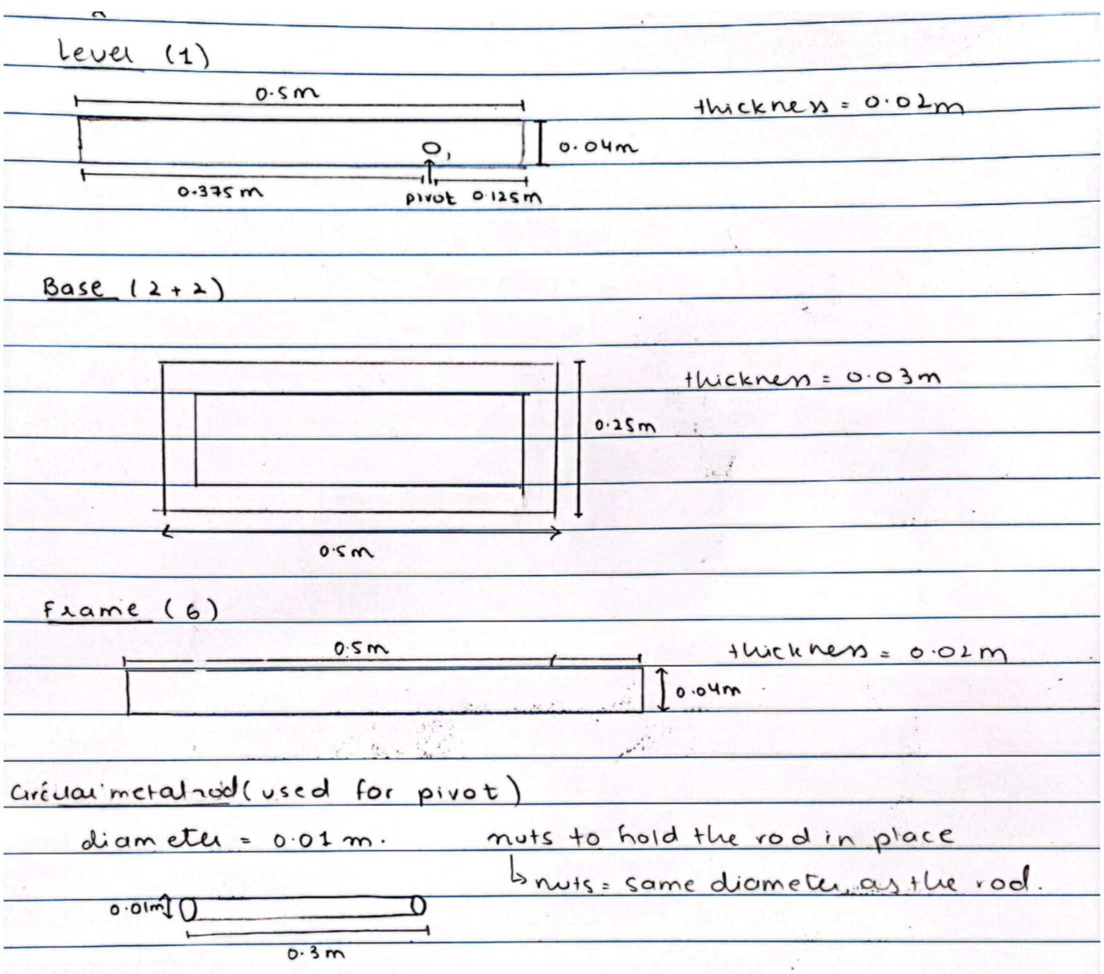
Side view



Front view



Measurements:



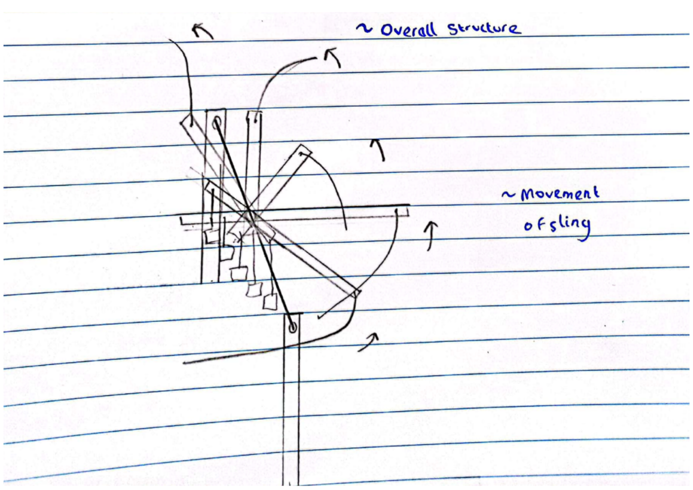
5 Experiment

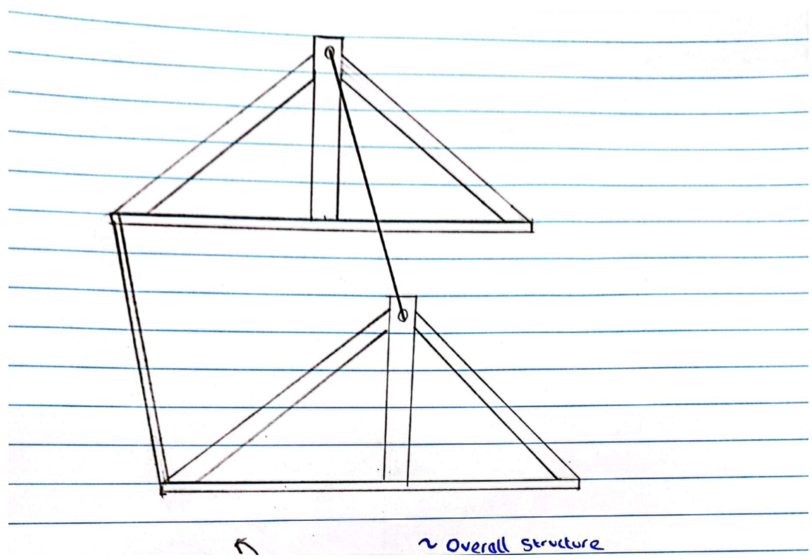
5.1 Building the Trebuchet

Material used

- Lever - one piece of rectangular wood (0.5 x 0.04 x 0.02)
- Base - three pieces of wood
  - 2 x (0.5 x 0.04 x 0.03)
  - 1 x (0.25 x 0.04 x 0.03)
- Frame - six pieces of wood (0.5 x 0.04 x 0.02)
- Pivot - Circular metal rod (diameter = 0.01 m, length = 0.3 m)
- Nuts to hold the rod in place (inner diameter = 0.01 m)
- String - 1m

Note: (The project was put together in the lab using technical help of professionals with wood cutting and assembling.)



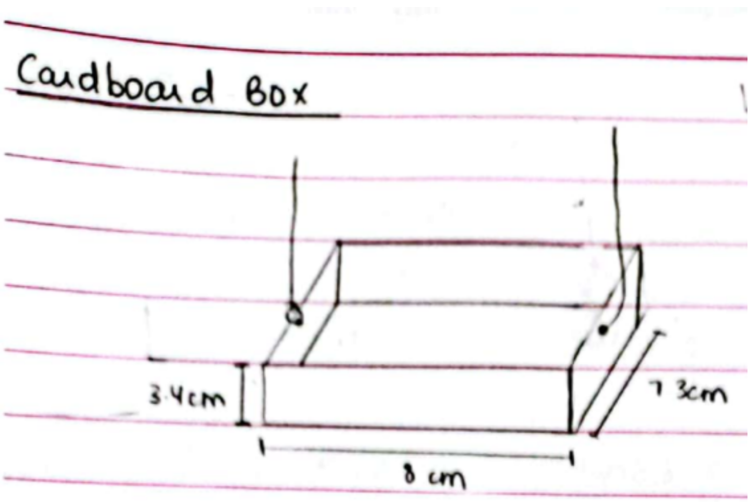


5.2 Testing with Different String Lengths and Payloads

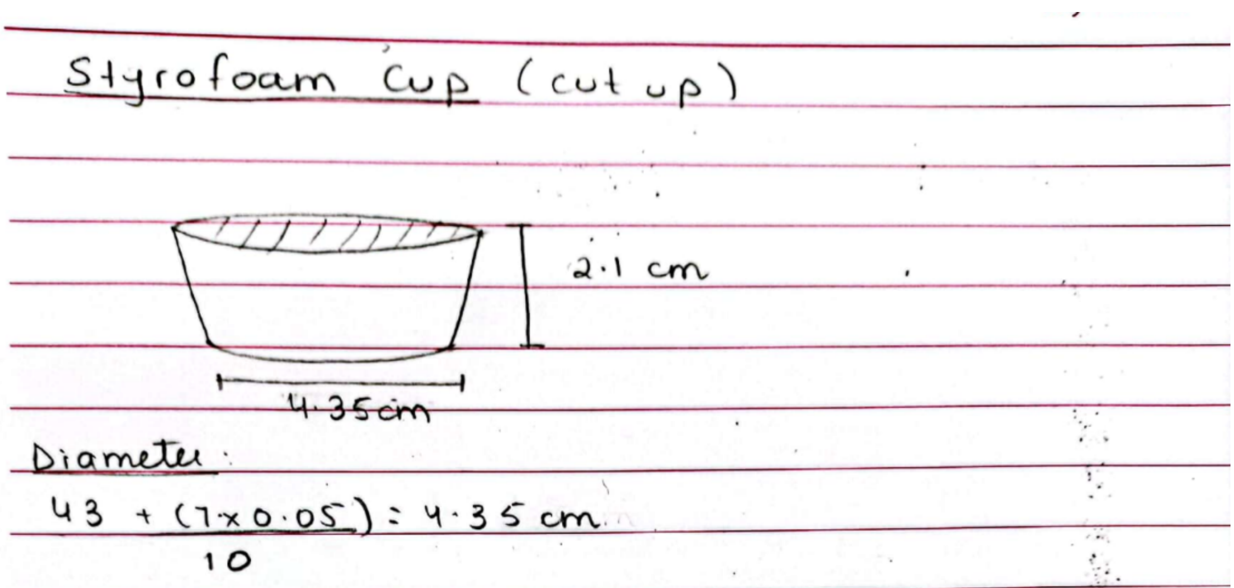
Two types:

1. Cardboard box

String length - 13 cm

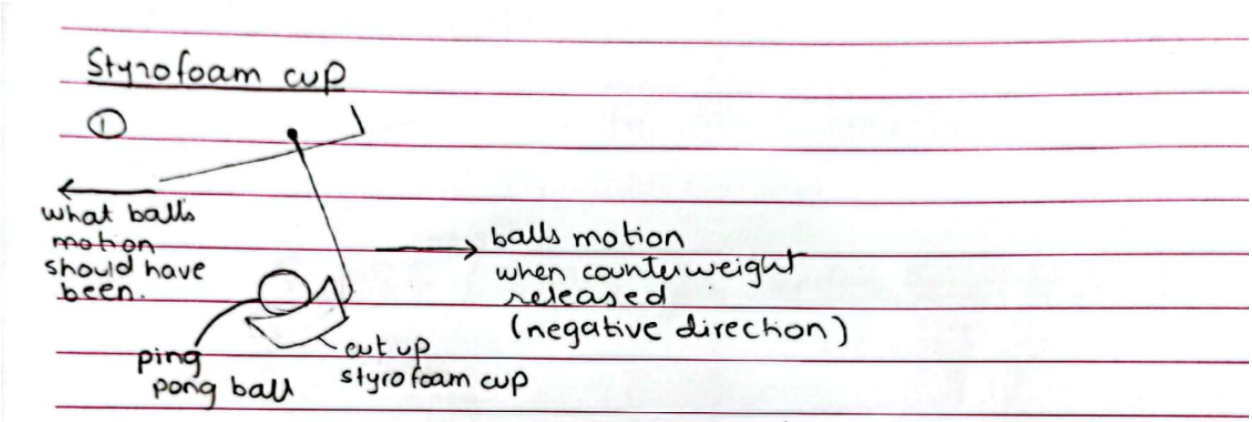


2. Cut up styrofoam cup



Case 1:

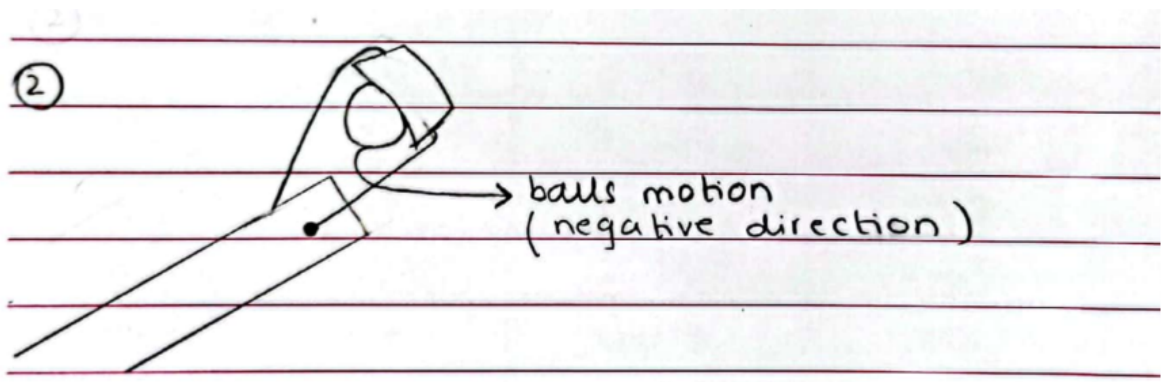
Attaching the string to just one side of the styrofoam cup. This resulted in the ball being leaving the cup as soon as it was released. The cup was unable to hold the ball and released it before the projectile could launch. The string used here was 25 cm long.



Case 2:

The string was attached to both ends of the cup. The ball was released at the top of the motion but it traveled in the opposite direction. The cup was able to hold the projectile but the centripetal force caused the ball to almost complete the circle and launch in the opposite direction of where the launch was intended.

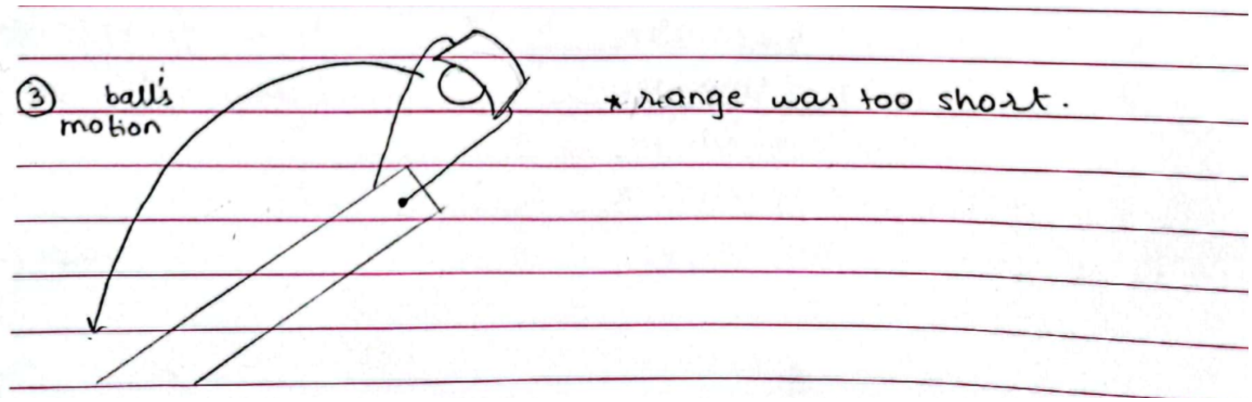
- The string is 25 cm long.



Case 3:

The string attached to the cup was made shorter and the ball was released at the right time and in the right direction but the range of the projectile was too short.

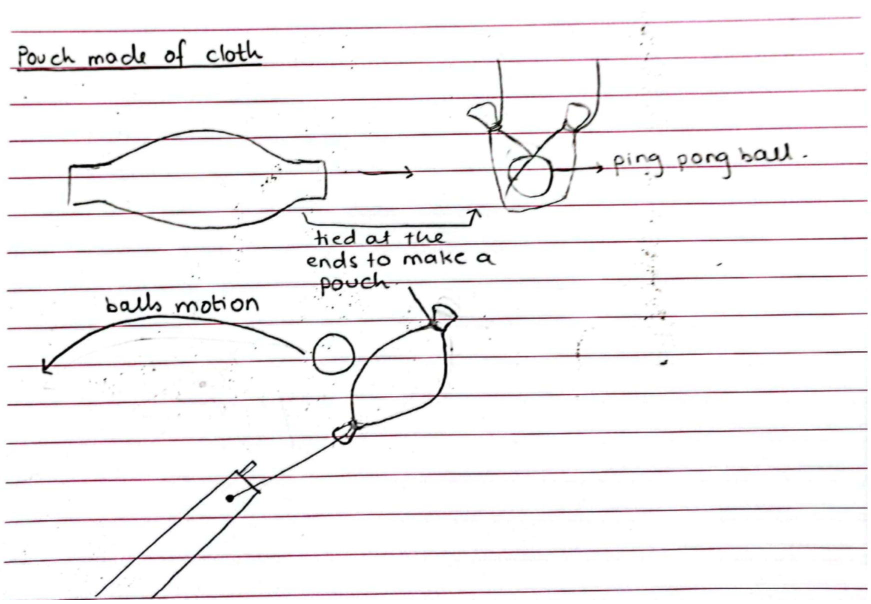
- The string is 15 cm long.



Final design which worked:

A pouch made of cloth is fixed at one end to the long arm and the other end is on a nail attached to the end of the long arm. When the weight is released, the string slips off the nail and the ball is launched, forming a projectile.

- The string attached to the long arm is 30 cm
- The string attached to the nail is 25 cm
- The cloth is 25 cm long.

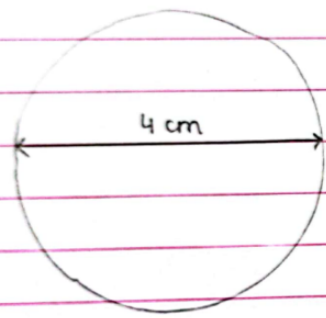


5.3 Testing with Variable Counterweights

We tested several different counterweights ranging from 300 g to 800 g. 300 g - 400 g did not give any significant range when the projectile was launched. Starting from 500 g to 800 g the range increased linearly.

5.4 Testing with Objects of Different Masses

Ping Pong Ball



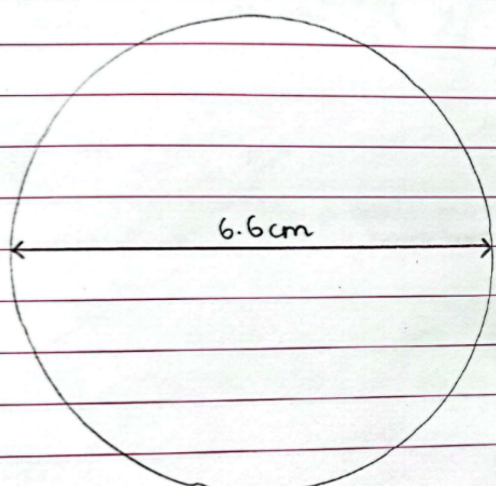
Mass = 2.8g

$$\text{Uncertainity\_mass} = \frac{0.1}{2\sqrt{3}} = 0.028 \simeq 0.03$$

(2.80 ± 0.03) g

Styrofoam Ball

$$\left(\frac{63}{10} + 6 \times 0.05 \right) = 6.6\text{cm}$$

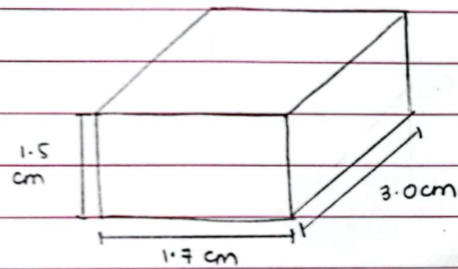


Mass = 3.0g

$$\text{Uncertainity\_mass} = \frac{0.1}{2\sqrt{3}} = 0.028 \simeq 0.03\text{g}$$

(3.00 ± 0.03) g

Eraser day / date:



Mass = 7.5 g

$$\text{uncertainity\_mass} = \frac{0.1}{2\sqrt{3}} = 0.028 \approx 0.03$$

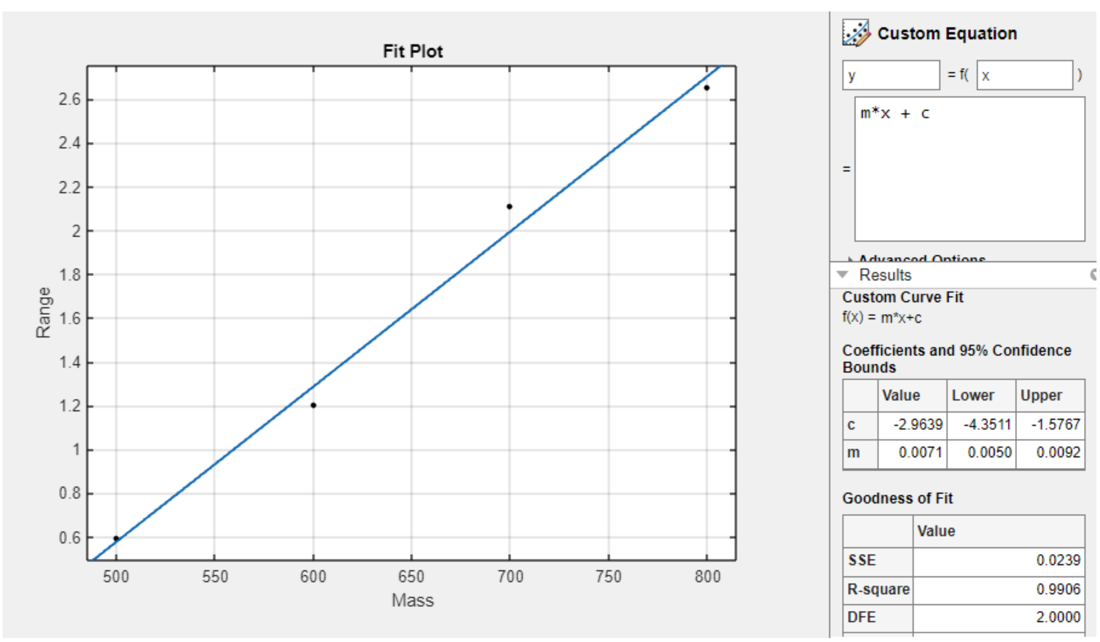
$$(7.5 \pm 0.03)\,g$$

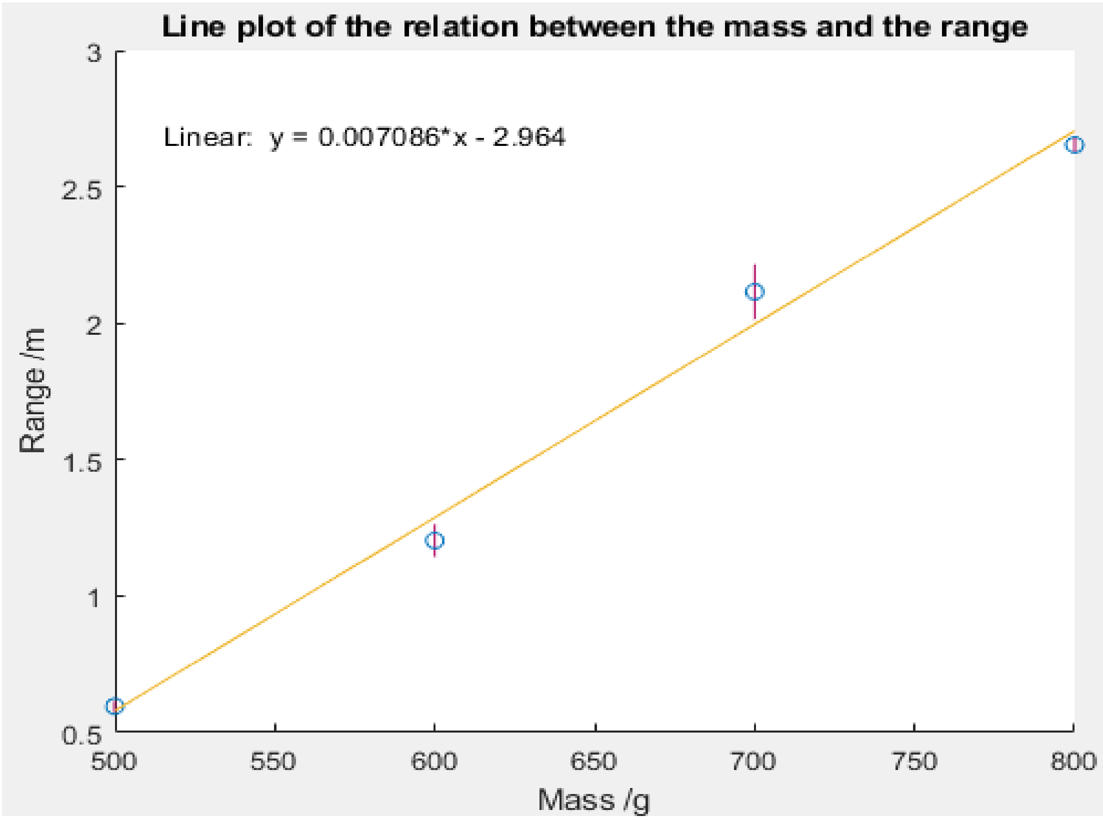
5.5 Plotting Graphs and Uncertainties

5.5.1 Variable Counterweight

<u>Experimental 1 data with variable counterweights :</u>
Keeping the object constant (ping pong ball)
Angle of the nail to the lever arm is approximately 90 degrees.
Angle of the launch is approximately 110 degrees.
For 300 g the projectile was not launching
For 400 g the range of the projectile was too short.

Distance/m	Reading 1/m	Reading 2/m	Reading 3/m	Reading 4/m	Reading 5/m
Mass/g					
300	–	–	–	–	–
400	–	–	–	–	–
500	0.501	0.598	0.667	0.585	0.627
600	1.472	1.263	1.065	1.103	1.116
700	2.581	2.264	1.874	1.979	1.875
800	2.603	2.661	2.594	2.713	2.699
Mass/g	500/g	600/g	700/g	800/g	
Average/m	0.5956	1.2038	2.1146	2.654	

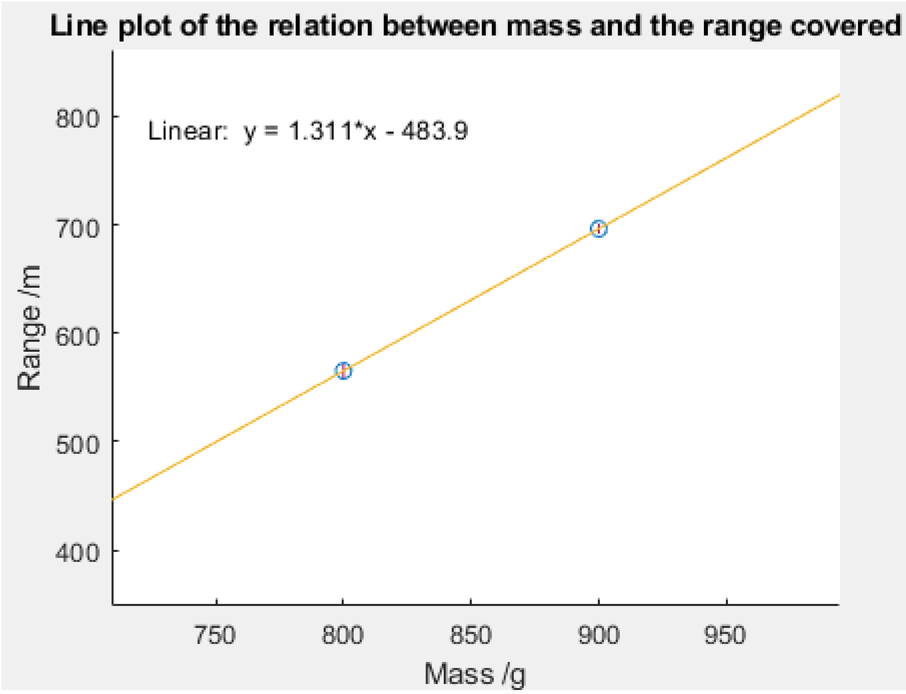




5.5.1 Variable Counterweight

<u>Experimental 2 data with variable counterweights:</u>
Keeping the object constant (ping pong ball)
Angle of the nail to the lever arm is approximately 75 degrees.
Angle of the launch is approximately 145 degrees.

Weights/g	R_1 /cm	R_2/cm	R_3 /cm	R_4 /cm	R_5/cm	Av/m
800	552.6	555.1	553.9	579.2	582.7	
meters →	5.526	5.551	5.539	5.792	5.827	5.647
900	683.9	707.6	701.1	703.8	687.6	
meters →	6.839	7.076	7.011	7.038	6.876	6.968




```
Mass = [800 900];

range800 = [555.2 555.1 553.9 579.2 582.7];
range900 = [683.9 707.6 701.6 701.1 687.6];

uncertanity = [(std(range800)/sqrt(5)) (std(range900)/sqrt(5))];
Range = [mean(range800, "all") mean(range900,"all")];

title('Line plot of the relation between mass and the range covered')
xlabel('Mass /g')
ylabel('Range /m')
xyerrorbar(Mass,Range,[0 0 0 0 ],uncertanity,'o')
```

5.5.2 Variable Object Mass

<u>Experimental data for different objects</u>

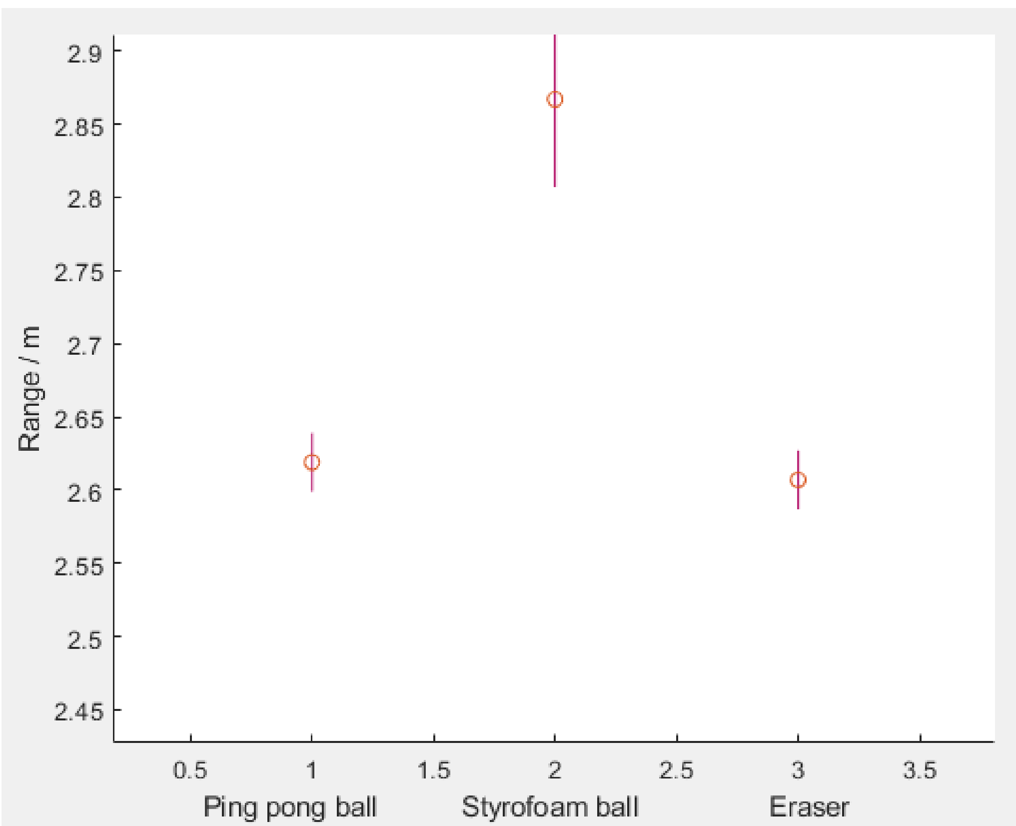
Keeping the counterweight constant (800 g)

Angle of the nail to the lever arm is approximately 90 degrees.

Angle of the launch is approximately 110 degrees.

Expeument 2 — Fixed counter weight mass = 800g

Distance/m	reading 1	reading 2	Reading 3	Average /m
Eraser	2.564	2.673	2.585	2.607
Ping pong	2.603	2.661	2.594	2.619
Styrofoam ball	2.760	3.010	2.830	2.867



6 Improvements

After testing we made a few changes which improved the launch angle and the range of the launched projectile.

We added a square block to the end of the short arm to increase the moment experienced by the weight so that the velocity of the launch increases.

We reduced the weight of the long arm by punching holes in it so that there were less obstructions in the motion of the launch.

We changed the angle of the nail from which the string had to slip off so that the launch angle, the height of launch and range of launch was improved.

We then took more readings with a variable counterweight and using a bearing ball as the object to be launched.

References

Ratio, dimensions and working of a trebuchet:

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https://youtu.be/9-Hwxw4fgqk?si=QBRWqjQ06XVTE4RU