PH4104 ASSIGNMENT 1

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Q1)

$\lambda_{red} = 700 \times 10^{-9} m$

$P = 23 \times 10^{-3} J/s$

$E = \frac{hc}{\lambda} = \frac{hc}{700 \times 10^{-9}}$] 1 red photon's Energy!

$\boxed{E = 2.837 \times 10^{-19}}$

Total Energy / Sec $= 23 \times 10^{-3}$

No of Photons $= \frac{\text{total Energy}}{\text{Energy of 1 photon}}$

$= \frac{23 \times 10^{-3}}{2.837 \times 10^{-19}} = \boxed{8.11 \times 10^{16} \text{ photons}}$

Q2] $z = 3+5i$, $y = 5+8i$

(a) $y \times z = (3+5i)(5+8i)$

$= \cancel{15 + 25i + 24i - 8}$

$= \cancel{15 + 24}$

$= 15 + 24i + 25i + 40i^2$

$= 15 - 40 + 49i$

$= \boxed{-25 + 49i}$

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(b) $z^* = 3 - 5i$

$y^* = 5 - 8i$

(c) $z + z^* = (3 - 5i) + (3 + 5i)$

$= 6$

$y + y^* = (5 - 8i)(5 + 8i)$

$= 10$

Real!

(d) $z - z^* = (3 + 5i) - (3 - 5i)$

$= \boxed{10i}$

$y - y^* = -(5 - 8i) + (5 + 8i)$

$= \boxed{16i}$

imaginary!

(e) $y \times y^* = (5 - 8i)(5 + 8i)$

$= 25 - 64i^2$

$= 25 + 64 = \boxed{89}$

$z \times z^* = (3 - 5i)(3 + 5i)$

$= 9 - 25i^2$

$= 9 + 25$

$= \boxed{34}$

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(f) $\frac{z}{y} = \frac{3+5i}{5+8i}$

$\frac{z}{y} = \frac{z}{y} \frac{y^*}{\times y^*} = \frac{(3+5i)(5-8i)}{(5+8i)(5-8i)}$

$= \frac{15-24i+25i-40i^2}{25-64i^2} = \frac{15+40+i}{25+64}$

$= \frac{55}{89} + \frac{i}{89}$

$\frac{y}{z} = \frac{y}{z} \times \frac{z^*}{z^*} = \frac{(5+8i)(3-5i)}{(3+5i)(3-5i)}$

$= \frac{15-25i+24i-40i^2}{9-25i^2} = \frac{55-i}{9+25}$

$= \frac{55}{34} - \frac{i}{34}$

(g) $|z| = \sqrt{a^2+b^2} = \sqrt{9+25} = \sqrt{34}$

$|y| = \sqrt{a^2+b^2} = \sqrt{25+64} = \sqrt{89}$

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Q3)

(a) $3+4i$

$R = \sqrt{a^2+b^2} = 5$

$\tan\theta = \dfrac{b}{a}$, $\theta = \tan^{-1}\left[\dfrac{3}{4}\right] = 36.9^\circ$
$= 0.64 \text{ rad}.$

$= 5e^{i(0.64)}$

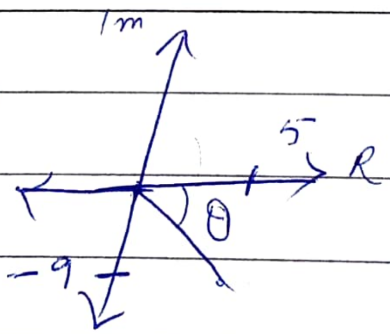
(b) $5+9i$

$R = \sqrt{5^2+9^2} = 10.3$

$\theta = \tan^{-1}\left[\dfrac{9}{5}\right] = 60.9^\circ = 1.06$

$= 10.3\, e^{i(1.06)}$

(c) $10.3e^{i\theta}$



$\theta = -60.9^\circ$

$= -1.06 \text{ rad}$

$10.3e^{-i(1.06)}$

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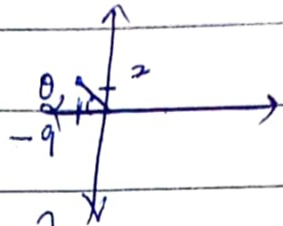
(d) $-9+2i$

$R=\sqrt{9^2+2^2}=9.21$

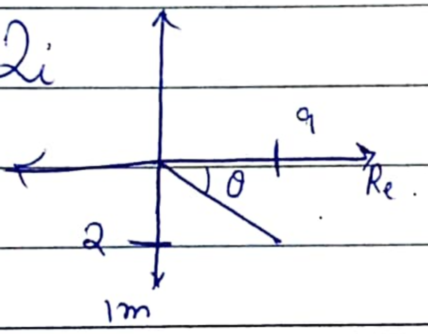
$\theta=\tan^{-1}\left[\frac{2}{-9}\right]$



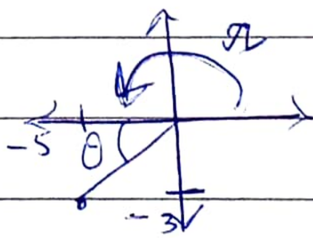
$-12.5^\circ=-0.22\,rad$

$\boxed{9.21e^{-i(0.22)}}$

(e) $9-2i$



$9.21e^{-i(0.22)}$



(f) $-5-3i$

$R=\sqrt{25+9}=5.83$

$\theta=\pi+0.54$

$5.83e^{i(\pi+0.54)}$

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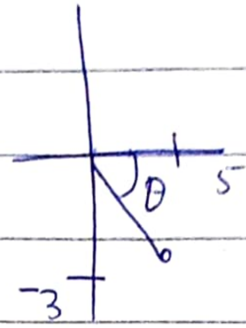
(g) $R = 5.83$

$\tan^{-1}\left[-\frac{3}{5}\right] = \theta$



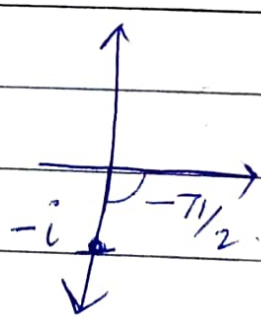
$5.83e^{-i(0.54)}$

(h) $\frac{1}{i} = \frac{1}{i}\left(\frac{-i}{-i}\right) = \frac{-i}{-i^2} = \frac{-i}{1}$

$0 - i$

$R = \sqrt{1^2} = 1$

$\theta = \tan^{-1}\left[\frac{-1}{0}\right] = \frac{-\pi}{2}.$



$1e^{\frac{-i\pi}{2}} = e^{-i\pi/2}$

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Q4) $Re^{i\theta} \rightarrow a+ib$ form.

(a) $5e^{i\pi/6}$.

$$Re^{i\theta} = R\cos\theta + iR\sin\theta$$

$$= \underbrace{5\cos\frac{\pi}{6}}_{a} + i\underbrace{5\sin\frac{\pi}{6}}_{b}.$$

$$= \frac{5}{2}\sqrt{3} + \frac{5}{2}i$$

(b) $9e^{i\pi} = 9\cos\pi + 9i\sin\pi$

$$= -9 + 0i$$

(c) $2e^{-i\pi/4} = 2\cos\frac{\pi}{4} - 2i\sin\frac{\pi}{4}.$

$$= \sqrt{2} - \sqrt{2}i$$

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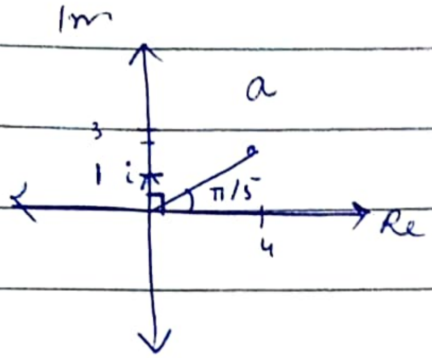
Q5) $5e^{i\pi/5} \times e^{i\pi/2} = 5e^{i\frac{\pi}{5} + i\frac{\pi}{2}}$

$=$



$Re^{i\theta} \to a+ib$ form.

$a: 5e^{i\pi/5} = 5\cos\frac{\pi}{5} + i5\sin\frac{\pi}{5}$

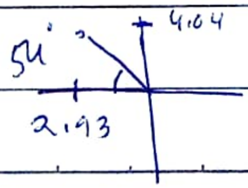
$= 4.04 + 2.9i$

$e^{i\pi/2} = b = 1\cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = 0 + i(\underline{1})$.

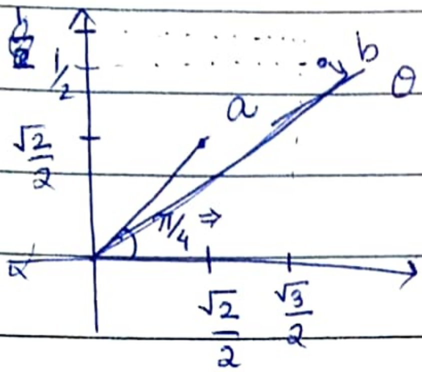
$5e^{i(\pi/2 + \pi/5)} = 5e^{i\frac{7}{10}\pi}$

$= 5\cos\frac{7}{10}\pi + i5\sin\frac{7}{10}\pi$

$= -2.93 + i4.04$



(b) $e^{i\pi/4} \times e^{i\pi/6}$



$\theta = \pi/6$

$a = e^{i\pi/4} = \cos\frac{\pi}{4} + i\sin \pi/4$

$= \sqrt{2}/2 + \sqrt{2}/2\, i$

(0.7) (0.7)

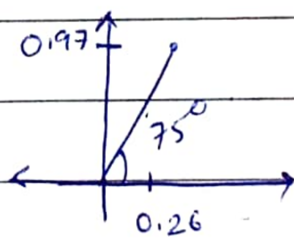
$b = e^{i\pi/6} = \cos \pi/6 + i \sin \pi/6$

$= \frac{\sqrt{3}}{2} + \frac{i}{2}$

$e^{i(\pi/4 + \pi/6)} = e^{i\frac{5}{12}\pi}$

$= \cos\frac{5}{12}\pi + i\sin\frac{5}{12}\pi$

$= 0.26 + 0.97i$



(c) Rotation! θ's add up!

(d) $|ze^{i\theta}|^2 = |z|^2$

$|z \times z \times \underbrace{e^{i\theta} \times e^{-i\theta}}| = |z|^2$

$|z^2 e^0| = |z|^2$

$|z|^2 = |z|^2$

Question 6

(a) Let's pass the input state through the interferometer.

$$
\begin{aligned}
|1\rangle &\xrightarrow{BS_1} \frac{1}{\sqrt{2}}\big(\,|0\rangle - |1\rangle\,\big) \\
&\xrightarrow{mirrors} \frac{1}{\sqrt{2}}\big(\,|1\rangle - |0\rangle\,\big) \\
&\xrightarrow{BS_2} \frac{1}{\sqrt{2}}\Big[\frac{1}{\sqrt{2}}\big(\,|0\rangle - |1\rangle\,\big) - \frac{1}{\sqrt{2}}\big(\,|0\rangle + |1\rangle\,\big)\Big] \\
&= -\,|1\rangle
\end{aligned}
$$

Therefore, the output state $|\psi_{out}\rangle = |1\rangle$. This means that the detector at the top clicks as we show now:

$$
\begin{aligned}
P(D_1) &= |\,\langle 0|\psi_{out}\rangle\,|^2 = 0 \\
P(D_2) &= |\,\langle 1|\psi_{out}\rangle\,|^2 \\
&= |-\langle 1|1\rangle\,|^2 = 1
\end{aligned}
$$

Notice that the minus sign in front of ψ_{out} is immaterial. It is called a global phase.

(b) If the input state were $\frac{1}{\sqrt{2}}\big(\,|0\rangle - |1\rangle\,\big)$, we have the following state propagator:

$$
\begin{aligned}
\frac{1}{\sqrt{2}}\big(\,|0\rangle - |1\rangle\,\big) &\xrightarrow{BS_1} \frac{1}{2}\Big(\,|0\rangle + |1\rangle - (|0\rangle - |1\rangle)\Big) \\
&= |1\rangle \\
&\xrightarrow{mirrors} |0\rangle \\
&\xrightarrow{BS_2} \frac{1}{\sqrt{2}}\big(\,|0\rangle + |1\rangle\,\big) = |\psi_{out}\rangle
\end{aligned}
$$

This means that the detector at the top clicks with a probability :

$$
\begin{aligned}
P(D_2) &= |\frac{1}{\sqrt{2}}\,\langle 1|\,(|0\rangle + |1\rangle)|^2 \\
&= \frac{1}{2}
\end{aligned}
$$

and similarly

$$\begin{aligned} P(D_1) &= |\frac{1}{\sqrt{2}}\langle 0| (|0\rangle + |1\rangle)|^2 \\ &= \frac{1}{2}. \end{aligned}$$

(c) i) Taking the input state at $|1\rangle$ and passing it through the interferometer with a blockage in one arm as

$$\begin{aligned} |1\rangle &\xrightarrow{BS_1} \frac{1}{\sqrt{2}}\big(\,|0\rangle - |1\rangle\,\big) \\ &\xrightarrow{mirrors} \frac{1}{\sqrt{2}}\big(\,|1\rangle - |0\rangle\,\big) \end{aligned}$$

The blocker will collapse the state of the photon to $|1\rangle$ with probability $\frac{1}{2}$. Hence, the state that enters the second beam splitter is given as

$$\begin{aligned} |1\rangle &\xrightarrow{BS_2} \frac{1}{\sqrt{2}}\big(\,|1\rangle - |0\rangle\,\big) \\ &= |\phi_{out}\rangle \end{aligned}$$

The output state has changed due to the presence of blocker and consequently the probabilities are given by:

$$\begin{aligned} P(D_1) &= \frac{1}{2}|\,\langle 0|\phi_{out}\rangle\,| = \frac{1}{4} \\ P(D_2) &= \frac{1}{2}|\,\langle 1|\phi_{out}\rangle\,| = \frac{1}{4} \end{aligned}$$

ii)The input state is now $\frac{1}{\sqrt{2}}\big(\,|0\rangle - |1\rangle\,\big)$.

$$\begin{aligned} \frac{1}{\sqrt{2}}\big(\,|0\rangle - |1\rangle\,\big) &\xrightarrow{BS_1} |1\rangle \\ &\xrightarrow{mirrors} |0\rangle \\ &\xrightarrow{Blocker} \textit{absorbs the photon} \end{aligned}$$

Therefore, none of the detectors click and their outcome probabilities are zero.

(d) In the first two parts, the probability adds up to one. In the subsequent parts, the probabilities of the detectors clicking do not add up to one because some of the photons are being absorbed by the blocker.

Question 7

(a) The difference between Q6 and Q7 is in the definition of the beam splitter. Beam-splitter can be made with either of these properties.

$$\begin{aligned} |0\rangle &\xrightarrow{BS_1} \frac{1}{\sqrt{2}}\big(\,|0\rangle + i\,|1\rangle\,\big) \\ &\xrightarrow{mirrors} \frac{1}{\sqrt{2}}\big(\,|1\rangle + i\,|0\rangle\,\big) \\ &\xrightarrow{BS_2} \frac{1}{2}\big[i\,|0\rangle + |1\rangle + i(|0\rangle + i\,|1\rangle)\big] = i\,|0\rangle \end{aligned}$$

Therefore, D_1 clicks with a 100 percent probability as we show:

$$\begin{aligned} P(D_1) &= |-i\,\langle 0|\,(i\,|0\rangle)|^2 \\ &= 1. \end{aligned}$$

Question 8

(a) $e^{i\theta} = 1 + i\theta + \frac{(i\theta)^2}{2!} + \frac{(i\theta)^3}{3!} + \frac{(i\theta)^4}{4!} + \frac{(i\theta)^5}{5!} + \cdots = 1 + i\theta - \frac{\theta^2}{2!} - i\frac{\theta^3}{3!} + \frac{\theta^4}{4!} + i\frac{\theta^5}{5!} + \cdots$

(b) $\cos(\theta) = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} + \cdots$

$\sin(\theta) = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \cdots$

(c) $\cos(\theta) + i\sin(\theta) = 1 + i\theta - \frac{\theta^2}{2!} - i\frac{\theta^3}{3!} + \frac{\theta^4}{4!} + i\frac{\theta^5}{5!} + \cdots = 1 + i\theta + \frac{(i\theta)^2}{2!} + \frac{(i\theta)^3}{3!} + \frac{(i\theta)^4}{4!} + \frac{(i\theta)^5}{5!} + \cdots$

(d) The results match, at least as far as the fifth power of θ. But how do we convince ourselves that this will continue to work? To do this, let's think about the pattern. It seems that terms with zero or even powers in θ that show up in the expansion of $e^{i\theta}$

show up in $\cos(\theta)$ and the rest show up in $\sin(\theta)$; so it would help to separate these two. This motivates us to write

$$e^{i\theta} = \sum_{n=0}^{\infty} \frac{(i\theta)^{2n}}{(2n)!} + \sum_{n=0}^{\infty} \frac{(i\theta)^{2n+1}}{(2n+1)!}.$$

Note that the first part bears zero and positive powers of θ, while the other part bears odd powers.

Using what we know about i (i.e. $i^2 = -1$) we can rewrite this as

$$e^{i\theta} = \sum_{n=0}^{\infty} \frac{(-1)^n(\theta)^{2n}}{(2n)!} + i\sum_{n=0}^{\infty} \frac{(-1)^n(\theta)^{2n+1}}{(2n+1)!}.$$

You may also look up the full Maclaurin series for $\cos(\theta)$ and $\sin(\theta)$ to see that the first term in this sum is $\cos(\theta)$ and that the second is $i\sin(\theta)$.

Note how writing out the expansions and working with a few terms gave us the insight to prove the general result; this is a constant theme in physics and math.

(e) You may see this link.

Question 9

(a) The possible values of potential energy are given below:

$$\begin{aligned} U &= -\boldsymbol{B} \cdot \boldsymbol{\mu} \\ &= -(gz)(\mu_s) \\ &= -(gz)(kS) \\ &= \mp(gz)(\frac{k\hbar}{2}) \\ &= \mp\left(\frac{gk\hbar}{2}\right)z. \end{aligned}$$

(b) $F = -\frac{dU}{dz} = \pm\left(\frac{gk\hbar}{2}\right)$. Note the derivative of the potential in the other two directions is zero, so this force is only in the z-direction.

(c) There would be no force if we used a homogeneous field since $\frac{dU}{dz} = 0$.

(d) The time ,t, a particle spends under the influence of the aforementioned force $= d1/v_0$.

The initial z component of the particle's velocity $= 0$.

The final z component of the particle's velocity, $v_z = \pm \left(\frac{gk\hbar}{2m}\right)\left(\frac{d1}{v_0}\right)$. This comes from the laws of motion, final velocity$=$ initial velocity $+$ $(\frac{F}{m})t$.

Hence, angle of deflection would become

$$\tan^{-1}\left(\frac{v_z}{v_0}\right)$$
$$\tan^{-1}\left(\pm\frac{gk\hbar(d1)}{2mv_0^2}\right).$$

(e) Three since we would now have three different values for the z component of the spin angular momentum.

Question 10

(a) A basis **spans** the vector space and is a **linearly independent** set of vectors.

By spanning, we mean that any vector in the space can be written by adding some combination of the basis vectors. For example, in the xy plane, I can write any vector using $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$. This means that for any vector $\boldsymbol{v}$, there exist some numbers a, b such that $\boldsymbol{v} = a\hat{\boldsymbol{x}} + b\hat{\boldsymbol{y}}$.

A set of vectors is linearly independent if any one vector in the set can not be written as a sum of some multiples of other vectors in the set. This property guarantees that when we use our basis to write out some vector, the combination we use to write it is unique. In our xy plane example, this means that for any vector $\boldsymbol{v}$, there exists **only one pair of numbers** a, b such that $\boldsymbol{v} = a\hat{\boldsymbol{x}} + b\hat{\boldsymbol{y}}$.

(b) $\langle i|i\rangle = 1$ for $i = 0, 1, 2, 3$.

(c) $\langle i|j\rangle = 0$ when $i \neq j$.

(d) We want $|\langle\psi|\psi\rangle|^2 = 1$ which implies (due to the orthonormal basis*) that $\left|\frac{1}{\sqrt{10}}\right|^2 + \left|\frac{3}{10}\right|^2 + \left|\frac{1}{\sqrt{10}}\right|^2 + |k|^2 = 1$. This implies that $|k|^2 = \frac{61}{100}$. Now k is a complex number whose magnitude squared we know. It could just be the real number $\sqrt{\frac{61}{100}}$, but it can also be $e^{i\theta}\sqrt{\frac{61}{100}}$ (see question 5).

(e) Probability of finding the system in the state $|0\rangle = |\langle 0|\psi\rangle|^2 = \left|\frac{1}{\sqrt{10}}\right|^2 = \frac{1}{10} = \frac{10}{100}$.

Probability of finding the system in the state $|3\rangle = |\langle 3|\psi\rangle|^2 = \left|e^{i\theta}\sqrt{\frac{61}{100}}\right|^2 = \frac{61}{100}$.

* We learned how to calculate numbers like $\langle 0|\psi\rangle$ in class, but didn't focus so much on $\langle\psi|\psi\rangle$. There are a few rules.

Let $|\psi\rangle = a\,|0\rangle + b\,|1\rangle$.

(a) $\langle\psi|\psi\rangle \geq 0$ for all $|\psi\rangle$.

(b) $\langle\psi|\psi\rangle = 0$ if and only if $|\psi\rangle = 0$.

(c) $\langle 0|\psi\rangle = \langle 0|\,(a\,|0\rangle + b\,|1\rangle) = a\,\langle 0|0\rangle + b\,\langle 0|1\rangle$.

(d) if $\langle\mu|\psi\rangle = z$, then $\langle\psi|\mu\rangle = z^*$.

You can use these to show that $\langle\psi|0\rangle = (a^*\,\langle 0| + b^*\,\langle 1|)\,|0\rangle = a^*\,\langle 0|0\rangle + b^*\,\langle 1|0\rangle$. Even if you don't understand anything from this discussion, make sure you are comfortable with this result! Try to use this final result to show that $\langle\psi|\psi\rangle = |a|^2 + |b|^2$ if $|0\rangle\,,|1\rangle$ form an orthonormal basis. By orthonormal, we mean that each basis vector is normalized and that the vectors are orthogonal to each other.

Of course, there is good reason to demand the four properties we have demanded. For example, (A) can be motivated by thinking about the familiar dot product in our xy plane. We also use the dot product of a vector with itself to talk about its "size". And we usually think of "size" as a real number. Can you see how rules (A) supports this behavior? Rule (D) is crucial in figuring out how to carry out $\langle\psi|0\rangle$ as we did above.

This is another theme in physics and math: we develop a feeling for what we want our tools to do which we use to come up with fundamental rules our tools must follow so they behave as desired.