Group 1 **Quiz 1** February 13, 2024

Name : Solution

Roll Number:

Question 1

In class, we studied the Mach-Zehnder interferometer. A figure of this setup can be found on the next page. The basis states are the sideward and upward propagating photon states and are represented by $|0\rangle$ and $|1\rangle$ respectively. In this problem, let's consider a slightly different operation of the beam splitters on the basis states to investigate its effect on the outcome probabilities of the quantum states. The operation of the first beam splitter, B1, is defined as:

$$|0\rangle \xrightarrow{B_1} a\,|0\rangle + ib\,|1\rangle$$
$$|1\rangle \xrightarrow{B_1} ib\,|0\rangle + a\,|1\rangle\,.$$

The operation of a different beam splitter, B2, is defined as:

$$|0\rangle \xrightarrow{B_2} b\,|0\rangle + ia\,|1\rangle$$
$$|1\rangle \xrightarrow{B_2} ia\,|0\rangle + b\,|1\rangle\,.$$

Note that $a, b \in \mathbb{R}$ and $a^2 + b^2 = 1$.

The two identical mirrors in the interferometer swap the two quantum states:

$$|0\rangle \xrightarrow{Mirrors} |1\rangle$$
$$|1\rangle \xrightarrow{Mirrors} |0\rangle\,.$$

(a) Suppose we input $|1\rangle$ into the interferometer. Show the progression of the quantum state till the very end, just before the detectors. [4]

(b) The probability of some detector D clicking is usually represented as $P(D)$. Find P(D1) and P(D2). [2]

(c) What happens if the path is blocked at location A? Repeat the previous part with this blocker. Explain how the probabilities sum up to 1. [6]

The rest of the question assumes that there is a blocker present.

(d) Which detector gives a no-interaction measurement? As in, which detector clicking tells you there is a blocker present? Explain your choice. [2]

(e) Calculate the efficiency of the no-interaction measurement. This is defined as

$$\frac{P(D)}{P(D) + P(\text{no detector clicks})}$$

where D is detector you picked in the previous part. [1]

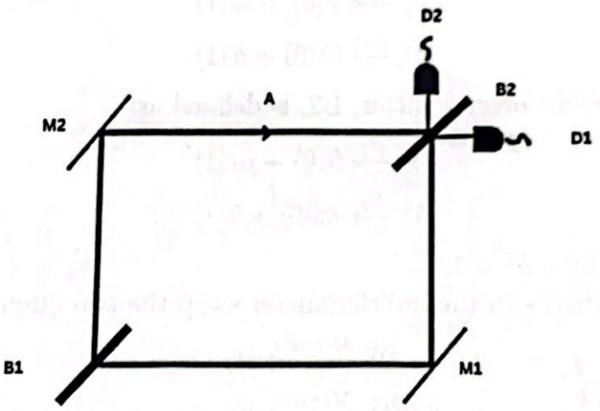

Figure 1: Schematic diagram for the Mach-Zehnder interferometer.

a^2 $|0\rangle$

b^2

1 (a) $|1\rangle \xrightarrow{B_1} ib|0\rangle + a|1\rangle$

$\xrightarrow{\text{Mirrors}} ib|1\rangle + a|0\rangle$

$\xrightarrow{B_2} ib\left(ia|0\rangle + b|1\rangle\right) + a\left(b|0\rangle + ia|1\rangle\right)$

$= \cancel{-ab}|0\rangle + ib^2|1\rangle + \cancel{ab}|0\rangle + a^2i|1\rangle$

$= (a^2+b^2)\,i\,|1\rangle$

(b) $P(D_1) = \left|\langle 0|\{-ab|0\rangle + ib^2|1\rangle + ab|0\rangle + a^2i|1\rangle\}\right|^2 = 0$

$= |-ab + ab|^2 = 0$

$P(D_2) = \left|\langle 1|\{a^2+b^2(i|1\rangle)\}\right|^2$

$= |i(a^2+b^2)|^2$ Given that $a^2+b^2=1$

$= |i|^2 = 1.$

(C) $|1\rangle \xrightarrow{B_1} ib|0\rangle + a|1\rangle$

$\xrightarrow{\text{Mirrors}} ib|1\rangle + a|0\rangle$

$\xrightarrow{\text{Blocker}}$ Absorbs $|0\rangle$ with Probability $|a|^2 = a^2$

- Therefore the state collapses to $|1\rangle$ with probability $|ib|^2 = b^2$

$|1\rangle \xrightarrow{B_2} ia|0\rangle + b|1\rangle$

$$P(D_1) = b^2 \left| \langle 0| \{ia|0\rangle + b|1\rangle\} \right|^2$$

$$= b^2 |ia|^2 = b^2|a|^2 = b^2a^2$$

$$P(D_2) = \left| \langle 1| \{ia|0\rangle + b|1\rangle\} \right|^2 b^2$$

$$= |b|^2 = b^2 \cdot b^2$$

$$= b^4$$

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Total probability =

$P(\text{Blocked}) + P(D_1) + P(D_2)$

$= a^2 + b^2 a^2 + b^4$

$= a^2 + b^2(a^2 + b^2)$

$= a^2 + b^2 = \underline{1}$.

(d) In part 2, we found that the Probability of D_1 clicking is zero. ~~Due~~ However, due to the presence of Blocker ~~$P(D_1) = \text{zero}$~~. $P(D_1) = b^2 a^2$. Therefore, D_1 ~~gives~~ can confirm the presence of the Blocker.

(e) $P(D) = P(D_1)$ (from previous part)

$= a^2 b^2$

$P(\text{no detector clicks}) = a^2$.

$$\text{efficiency} = \frac{a^2 b^2}{a^2 b^2 + a^2}$$

$$= \frac{\cancel{a^2} b^2}{\cancel{a^2}(b^2+1)}$$

$$= \frac{b^2}{b^2+1}.$$