Group 2	**Quiz 1**	PHY 104 February 13, 2024

Name : Solution

Roll Number:

Question 1

In class, we studied the Mach-Zehnder interferometer. In this question, we will work with a Michelson interferometer; a figure of this setup can be found on the next page. Just like in class, a photon has two basis states, $|0\rangle$ and $|1\rangle$, corresponding to whether it takes a left-right path or an up-down path respectively. The photon enters in the state $|0\rangle$, sees the beam splitter B, reflects from the mirrors M1, M2, and then enters the beam splitter again. The action of the beam splitter is given below.

$$|0\rangle \xrightarrow{B} \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$$
$$|1\rangle \xrightarrow{B} \frac{i}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle .$$

The actions of M1 and M2 are identical. It is given below.

$$|0\rangle \xrightarrow{Mirrors} |0\rangle$$
$$|1\rangle \xrightarrow{Mirrors} |1\rangle .$$

(a) With a quantum state propagation approach, find the probability that detector D fires. [5]

(b) What if block the path at point A? How does the probability calculated above change? [4]

(c) Describe how the probabilities with the blocker in place add up to 1. [2]

(d) Suppose instead of a blocker, a phase element,ϕ, is placed at location A. Its action is given below.

$$|0\rangle \xrightarrow{\phi} e^{i\phi}|0\rangle$$
$$|1\rangle \xrightarrow{\phi} |1\rangle .$$

Calculate the probability that D clicks now. [5]

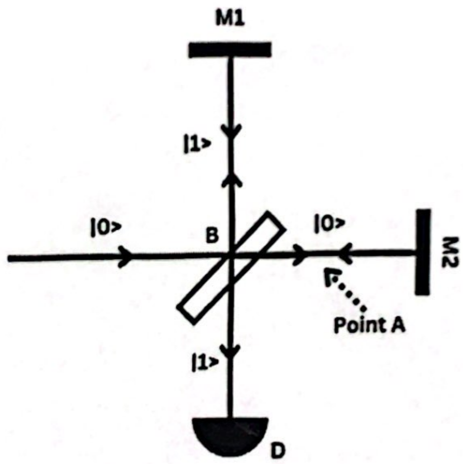

Figure 1: Schematic diagram for a Michelson interferometer.

a) $|0\rangle \xrightarrow{B} \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$

$\xrightarrow{M} \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$

$\xrightarrow{B} \frac{1}{\sqrt{2}}\left(\frac{|0\rangle + i|1\rangle}{\sqrt{2}}\right) + \frac{i}{\sqrt{2}}\left(\frac{i|0\rangle + |1\rangle}{\sqrt{2}}\right)$

$= \frac{1}{2}(|0\rangle + i|1\rangle) + \frac{(-1)}{2}|0\rangle + \frac{i|1\rangle}{2}$

$= \frac{i}{2}|1\rangle + \frac{i}{2}|1\rangle = i|1\rangle$

D detects $|1\rangle$

$P(D) = |\langle 1|(i|1\rangle)|^2$

$= |i\langle 1|1\rangle|^2$

$= |i|^2 = 1$

b) $|0\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|0\rangle + \frac{i}{\sqrt{2}}|1\rangle\right)$

Blocker blocks $|0\rangle$.

With $\left|\frac{i}{\sqrt{2}}\right|^2 = \frac{1}{2}$ probability, $|1\rangle$ survives.

So $|1\rangle$

$$|1\rangle \xrightarrow{M} |1\rangle \xrightarrow{B} \frac{i|0\rangle + |1\rangle}{\sqrt{2}}$$

D detects $|1\rangle$.

$$P(D) = \left|\langle 1| \left(\frac{i|0\rangle + |1\rangle}{\sqrt{2}}\right)\right|^2 \left(\frac{1}{2}\right)$$

$$\cancel{\left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2} + P(D)}$$

$$= \left|\frac{1}{\sqrt{2}}\right|^2 \left(\frac{1}{2}\right) = \frac{1}{4} = P(D)$$

c) $P(\text{absorption}) + P(D) = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$.

This leaves a $\frac{1}{4}$ probability we are yet to explain. This is associated with the photon being reflected by the beamsplitter, back to its source.

~~d)~~

(d) $|0\rangle \xrightarrow{B} \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$

$\xrightarrow{\phi} \frac{1}{\sqrt{2}}e^{i\phi}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$

$\xrightarrow{M} \frac{1}{\sqrt{2}}e^{i\phi}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$

$\xrightarrow{\phi} \frac{1}{\sqrt{2}}e^{i2\phi}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$

$\xrightarrow{B} \frac{1}{\sqrt{2}}e^{i2\phi}\left[\frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle\right]$

$+$

$\frac{i}{\sqrt{2}}\left[\frac{i}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right]$

$= \frac{1}{2}e^{i2\phi}\left[|0\rangle + i|1\rangle\right]$

$+$

$\frac{1}{2}\left[-|0\rangle + i|1\rangle\right]$

$= \left[\frac{e^{i2\phi}}{2} - \frac{1}{2}\right]|0\rangle + \left[\frac{i}{2}e^{i2\phi} + \frac{i}{2}\right]|1\rangle = |out\rangle$

$$P(D_1) =$$

$$|\langle 1 | out \rangle|^2$$

$$= \left| \frac{i}{2} e^{i2\phi} + \frac{i}{2} \right|^2$$

$$= \left| \frac{i}{2} (e^{i2\phi} + 1) \right|^2$$

$$= \left|\frac{i}{2}\right|^2 \left| e^{i2\phi} + 1 \right|^2 = \frac{1}{4} \left| e^{i2\phi} + 1 \right|^2$$

$$= \frac{1}{4} (1 + e^{i2\phi})(1 + e^{-i2\phi})$$

$$= \frac{1}{4} (1 + e^{i2\phi} + e^{-i2\phi} + 1)$$

$$= \frac{1}{4} (2 + 2\cos(2\phi))$$

$$= \frac{1}{2} (1 + \cos(2\phi))$$