PHY104 - Midterm Exam

Each question carries the same mark. There are 13 questions.

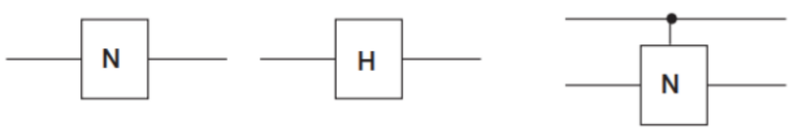
Fill in the multiple-choice answer sheet with a pencil.

You can only pick one option per question

Your Variant Code: 20. Write this on your multiple-choice answer sheet.

For this exam, we use the following conventions.

$$|D\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$
$$|A\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$
$$|L\rangle = \frac{|0\rangle + i\,|1\rangle}{\sqrt{2}}$$
$$|R\rangle = \frac{|0\rangle - i\,|1\rangle}{\sqrt{2}}$$



Input	Output
$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$

Input	Output
$\|0\rangle$	$\frac{1}{\sqrt{2}}(\|1\rangle + \|0\rangle)$
$\|1\rangle$	$\frac{1}{\sqrt{2}}(\|1\rangle - \|0\rangle)$

Input	Output
$\|0\rangle\|0\rangle$	$\|0\rangle\|0\rangle$
$\|0\rangle\|1\rangle$	$\|0\rangle\|1\rangle$
$\|1\rangle\|0\rangle$	$\|1\rangle\|1\rangle$
$\|1\rangle\|1\rangle$	$\|1\rangle\|0\rangle$

$$|\psi\rangle = a\,|0\rangle + b\,|1\rangle = \begin{pmatrix} a \\ b \end{pmatrix}.$$

$$|\psi\rangle = a\,|00\rangle + b\,|01\rangle + c\,|10\rangle + d\,|11\rangle = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}.$$

For three qubit states, the state $a\,|000\rangle + b\,|001\rangle + c\,|010\rangle + d\,|011\rangle + e\,|100\rangle + f\,|101\rangle + g\,|110\rangle + h\,|111\rangle$ **can be written as a column vector with entries in the same order as they are written out.**

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1. Which of the following states is **not** entangled?

   A. $\frac{1}{\sqrt{2}}(|000\rangle + i\,|111\rangle)$

   B. $\frac{1}{\sqrt{2}}(|0000\rangle - |1111\rangle)$

   C. $\frac{1}{\sqrt{3}}\,|00\rangle + \sqrt{\frac{2}{3}}\,|01\rangle$

   D. $\frac{1}{\sqrt{2}}\,(|01\rangle - |10\rangle)$

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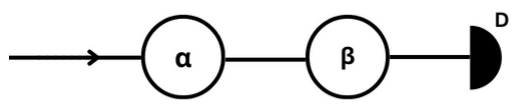
2. A photon in the polarizer state $|\varphi\rangle = |H\rangle$ sees two polarizers whose optic axes are at α and β, respectively, to the horizontal. The polarizers create a new quantum state whose polarization is parallel to the optic axes of polarizer. Detector D detects the presence of photons (in any polarisation). What is the probability that D clicks?



A. $cos^2(\alpha)cos^2(\beta)$ B. 0 C. $cos^2(\alpha)cos^2(\beta+\alpha)$ D. $cos(\alpha)cos(\beta)$ E. $cos^2(\alpha)cos^2(\beta-\alpha)$

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3. Alice has a qubit in the state $a\,|0\rangle + b\,|1\rangle$. She would like to convert this to $a\,|0\rangle + ib\,|1\rangle$. She should apply which of the following gates?

   A. $\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$

   B. $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$

   C. $\begin{pmatrix} 0 & 0 \\ 0 & i \end{pmatrix}$

   D. $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

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4. A Bell state $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ is measured. The first qubit is measured along the basis $\{|0\rangle\,, |1\rangle\}$. While the second qubit is measured along the basis $\{|D\rangle\,, |A\rangle\}$. What is the probability that the output state is $|0\rangle\,|D\rangle$?

   A. 0.5

   B. 0.25

   C. 0

   D. 0.25i

   E. 1

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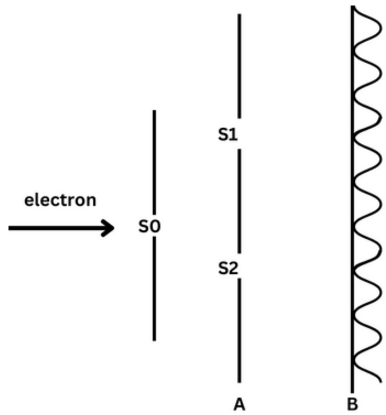
5. Electrons are fired onto a screen, one by one. An electron sees a slit S0 followed by a pair of slits S1 and S2. The electron then falls onto a screen.



Which of the following statements about this experiment is **correct**?

A. The electron is in a superposition of states between A and B.

B. The electron splits between A and B.

C. The electron's quantum state is undefined between A and B.

D. We know which slit the electron has passed through.

E. The single electron falls at many locations on the screen in one go.

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6. The first qubit in the entangled state

$$\frac{|01\rangle + |10\rangle}{\sqrt{2}},$$

is measured.

If the outcome is $|0\rangle$, what can be said about the second qubit's measurement (in the same basis)?

A. The output could be $|0\rangle$ or $|1\rangle$ with equal probability.

B. The output will most definitely be $|0\rangle$.

C. The output is $|1\rangle$ with probability of $\frac{1}{2}$.

D. The output is $|0\rangle$ with probability of $\frac{1}{2}$.

E. The output will most definitely be $|1\rangle$.

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7. A quantum state is $|\varphi\rangle = a|0\rangle + e^{i\phi}b|1\rangle$. The state is fed into an analyzer which has two output detectors corresponding to $|0\rangle$ and $|1\rangle$ outputs. The $|1\rangle$ detector clicks $\frac{1}{3}$ times, for identically prepared input states. Which of the following statements can be **true**?

   A. $a = 1 - \frac{1}{3}$, $b = \sqrt{\frac{1}{3}}$

   B. $a = (1 - \frac{1}{3})$, $b = \frac{1}{3}$

   C. $a = \sqrt{\frac{2}{3}}$, $b = \sqrt{\frac{1}{3}}$

   D. $a = \frac{2}{3}$, $b = \frac{1}{3}$

   E. $a = \sqrt{1 - (\frac{1}{3})^2}$, $b = (\frac{1}{3})^2$

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8. In the quantum state $|\psi\rangle = a|0\rangle + b|1\rangle$, what do the numbers a, b represent?

   A. They are merely used to normalize the state.

   B. Their sum must be one.

   C. They carry no meaning on their own.

   D. The probabilities of detecting the state in $|0\rangle$ and $|1\rangle$, respectively.

   E. Their modulus squares give the probabilities of detecting the state in $|0\rangle$ and $|1\rangle$, respectively.

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9. A pair of photons is produced in the state $\frac{1}{\sqrt{3}}|HH\rangle + \sqrt{\frac{2}{3}}|VV\rangle$. One photon goes to Bob. He measures along the $|H\rangle$ or $|V\rangle$ axis. Which of the following statements for Bob is **true**?

   A. $P(|H\rangle) = \frac{1}{\sqrt{3}}$ and $P(|V\rangle) = \sqrt{\frac{2}{3}}$.

   B. $P(|H\rangle) = 1/2$ and $P(|V\rangle) = 1/2$.

   C. $P(|H\rangle) = 1$ and $P(|V\rangle) = 0$.

   D. $P(|H\rangle) = 1/3$ and $P(|V\rangle) = 2/3$.

   E. $P(|H\rangle) = 0$ and $P(|V\rangle) = 1$.

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10. A qutrit is a three-level quantum system. Its (orthonormal) basis vectors are $|a\rangle$, $|b\rangle$, and $|c\rangle$. Which of the following is **not** a legitimate qutrit state?

   A. $\frac{|a\rangle+|b\rangle+|c\rangle}{\sqrt{2}}$

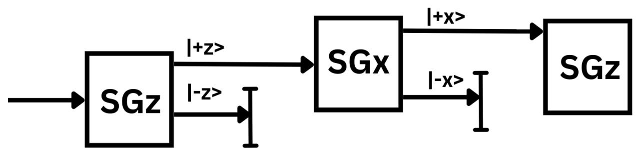
   B. $\frac{|a\rangle+|b\rangle+|c\rangle}{\sqrt{3}}$

   C. $\frac{1}{2}|a\rangle + \sqrt{\frac{3}{8}}|b\rangle + \sqrt{\frac{3}{8}}|c\rangle$

   D. $\frac{1}{2}|a\rangle + \sqrt{\frac{3}{8}}|b\rangle + e^{i\frac{\pi}{3}}\sqrt{\frac{3}{8}}|c\rangle$

   E. All of the given states are legitimate.

11. Electrons in some quantum state are passed through a Stern-Gerlach setup with magnets aligned in the z-direction. Two beams are produced in $|+z\rangle$ and $|-z\rangle$ states. The $|-z\rangle$ electrons are blocked, and the remaining are passed through magnets aligned in the x-direction. Again, only electrons found in the state $|+x\rangle$ survive, and the remaining are blocked. The electrons that survive are again input to magnets aligned in the z-direction as shown. The output from the last magnets is not shown. Which of the following is **correct**?



A. At the end, all the electrons are found in the state $|+z\rangle$.

B. At the end, some electrons are found in the state $|+z\rangle$ and some in $|-z\rangle$.

C. At the end, all the electrons are found in the state $|-z\rangle$.

D. At the end, none of the electrons are found in $|+z\rangle$ or $|-z\rangle$ states.

E. None of the statements above are correct.

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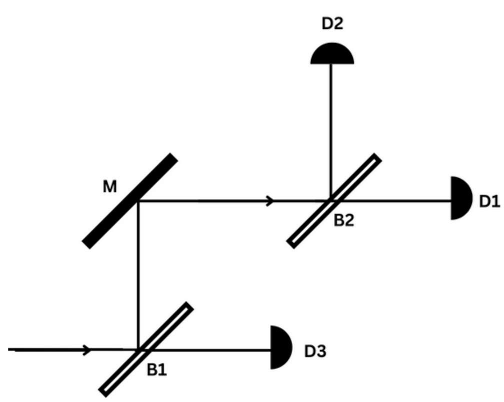
12. Consider the given arrangement of 50:50 beamsplitters (B1 and B2) and a perfect mirror (M).



$|H\rangle$ is input to B1. What are the probabilities that the detectors click?

A. P(D1)=1/2, P(D2)=1/2, P(D3)=0

B. P(D1)=1/4, P(D2)=1/4, P(D3)=1/2

C. P(D1)=0, P(D2)=0, P(D3)=1

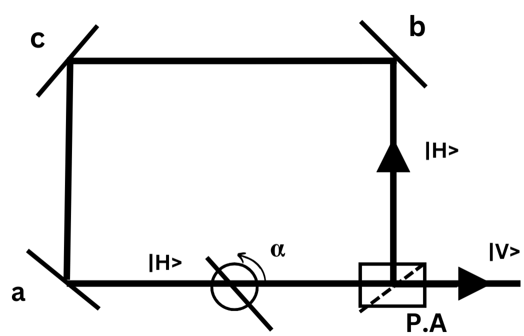
D. P(D1)=1/3, P(D2)=1/3, P(D3)=1/3

E. P(D1)=1/4, P(D2)=1/2, P(D3)=1/4

13. A photon is trapped in a ring, which is bounded by three perfect mirrors a, b, and c that merely change the photon's direction. A polarisation analyser has two output ports, a $|V\rangle$ for transmission and $|H\rangle$ for reflection. If we input $a\,|H\rangle + b\,|V\rangle$ into the analyser, it outputs a photon in the state $|H\rangle$ port with probability $|a|^2$ and in the $|V\rangle$ port with probability $|b|^2$.

A photon with polarization $|H\rangle$ is injected into the ring just to the right of mirror a as shown in the figure. It sees a polarization rotator that rotates $|H\rangle$ towards $|V\rangle$ by an angle α. What is the probability that the photon emerges from the $|V\rangle$ channel of the beam splitter after fully completing **N** cycles? The photon traverses the following trajectory in one cycle of the given quantum circuit:

$$a \mapsto \text{rotator} \mapsto \text{Polarisation Analyser} \mapsto \text{b} \mapsto \text{c} \mapsto \text{a}$$



A. $\sin^2(\alpha)cos^2(\alpha)$

B. $\sin^2(\alpha)$

C. $\sin^{2N}(\alpha)cos^{2N}(\alpha)$

D. $\sin^2(\alpha)cos^{2N}(\alpha)$

E. $\sin^{2N}(\alpha)cos^2(\alpha)$

Mid Solution

Question 1

(C) state: $\frac{1}{\sqrt{3}}|00\rangle + \sqrt{\frac{2}{3}}|01\rangle$

$$\rightarrow |0\rangle \otimes \left(\frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle \right)$$

Separable state and no correlation between the first and second qubit.

Question 2

(E) ① $\left| \left(\cos\alpha \langle H| + \sin\alpha \langle V| \right) |H\rangle \right|^2$

$= \cos^2\alpha$

↑

Probability through the first polarizer.

② $\left| \left(\cos\alpha \langle H| + \sin\alpha \langle V| \right) \left(\cos\beta |H\rangle + \sin\beta |V\rangle \right) \right|^2$

$= \left| \cos\alpha \cos\beta + \sin\alpha \sin\beta \right|^2$

Using trig. identity

$= \left| \cos(\alpha - \beta) \right|^2$

$= \cos^2(\alpha - \beta)$

Total probability is given by $= \cos^2\alpha \cos^2(\alpha - \beta)$

Question 3 (A)

We want to add a phase 'i' to the second qubit only.

The matrix for that is a 2x2 matrix given by

$$\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

Let's verify

$$\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ ib \end{pmatrix}$$

Question 4 (B)

$$|0\rangle|0\rangle \rightarrow |0\rangle \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$= \frac{1}{\sqrt{2}}(|00\rangle + |01\rangle)$$

$$P = \left| \frac{1}{\sqrt{2}}(\langle 00| + \langle 01|) \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \right|^2$$

$$= \left|\frac{1}{2}\right|^2 = 0.25$$

Question 5 *

(A) A superposition of state is created.

Question 6* (E)

If the first qubit is measured in $|0\rangle$ then the second qubit must be in $|1\rangle$ state

$$|01\rangle$$

↑ 1st ↘ 2nd

Question 7 (C)

$$|\phi\rangle = a|0\rangle + e^{i\phi} b|1\rangle$$

$$|\langle 1|\phi\rangle|^2 = \frac{1}{3} = |b|^2$$

$$\frac{1}{\sqrt{3}} = b$$

Therefore

$$|\langle 0|\phi\rangle|^2 = \frac{2}{3} = |a|^2$$

$$a = \sqrt{\frac{2}{3}}$$

Question 8

(E) a and b modulus square gives us the probabilities of detecting the state in $|0\rangle$ and $|1\rangle$, respectively.

Question 9 (D)

Here we make an assumption, if Bob detects $|H\rangle$ then the other qubit should also be in $|H\rangle$ state and so on.

Hence, we are talking about the probability of detecting $|HH\rangle$.

Question 10 (A)

Lets check for each case ⇒ We want $|a|^2 + |b|^2 + |c|^2 = 1$ second

A. $\left|\frac{1}{\sqrt{2}}\right|^2 + \left|\frac{1}{\sqrt{2}}\right|^2 + \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2} \neq A$

↑ Not a legitimate state.

Use similar method to verify for other states.

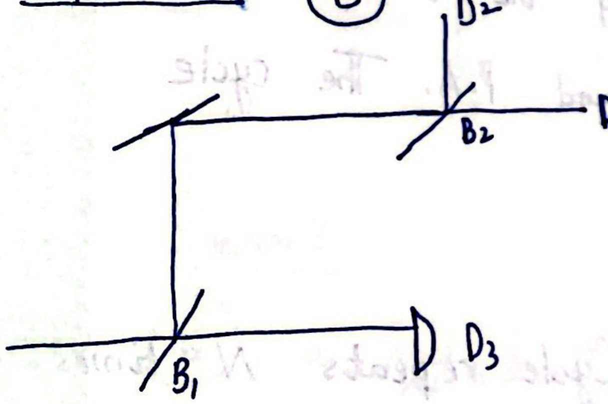
Question 11

(A) → Done in class

After passing $|+z\rangle$ throug SG_x we ~~lose the~~ create a new quantum state and so we can't say that it's still in $|+z\rangle$ state.

a|²

Question 12 (B)



With ½ probability $|V\rangle$ goes to the mirror of

↓

$|V\rangle \xrightarrow{\text{Mirror}} |H\rangle$

$|H\rangle \xrightarrow{B_1} \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$

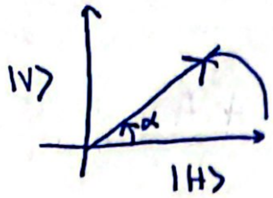
$|H\rangle \rightarrow \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$ n

$P(D_3) = \left|\langle H| \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)\right|^2 = \frac{1}{2}.$ in.

$P(D_1) = \frac{1}{4}$ ctin

$P(D_2) = \frac{1}{4}$

Question 12



Rotator $|H\rangle \rightarrow \sin\alpha|V\rangle + \cos\alpha|H\rangle$

$\sin\alpha|V\rangle + \cos\alpha|H\rangle \xrightarrow{P.A}$

Reflection probability is given by

$$\left|\langle H|\left(\sin\alpha\,|V\rangle + \cos\alpha\,|H\rangle\right)\right|^2$$

$$= \cos^2\alpha$$

Transmission " "

$$\left|\langle V|\left(\sin\alpha\,|V\rangle + \cos\alpha\,|H\rangle\right)\right|^2$$

$$= \sin^2\alpha$$

Now $|H\rangle$ will be reflected by the three mirrors and again enter Rotator and P.A. The cycle repeats.

~~$(\cos^2\alpha)^N \cdot \sin^2\alpha$~~

~~$= \cos^{2N}\alpha \cdot \sin^2\alpha$~~

Cycle repeats N times and the probability is given by $(\cos^2\alpha)^N = \cos^{2N}\alpha$

After N cycles, prob of gettive $|V\rangle$ is $\left(\sin^2\alpha \cdot \cos^{2N}\alpha\right)$