Question 1

Given the wavefunction of a particle:

$$\psi(x) = \frac{Ax}{b}\exp\left\{-\frac{x^2}{2b^2}\right\}.$$

(a) Plot the wavefunction $\psi(x)$.

(b) Plot the probability density function for the given $\psi(x)$.

(c) What is the probability of finding the particle between $x = 0$ and $x = \frac{b}{2}$?

(d) Where is the probability density the highest?

Question 2

The potential energy graph for a molecule is shown below. Find the shortest wavelength of light that can be absorbed by the molecule, given that the ground state energy is $-20eV$.

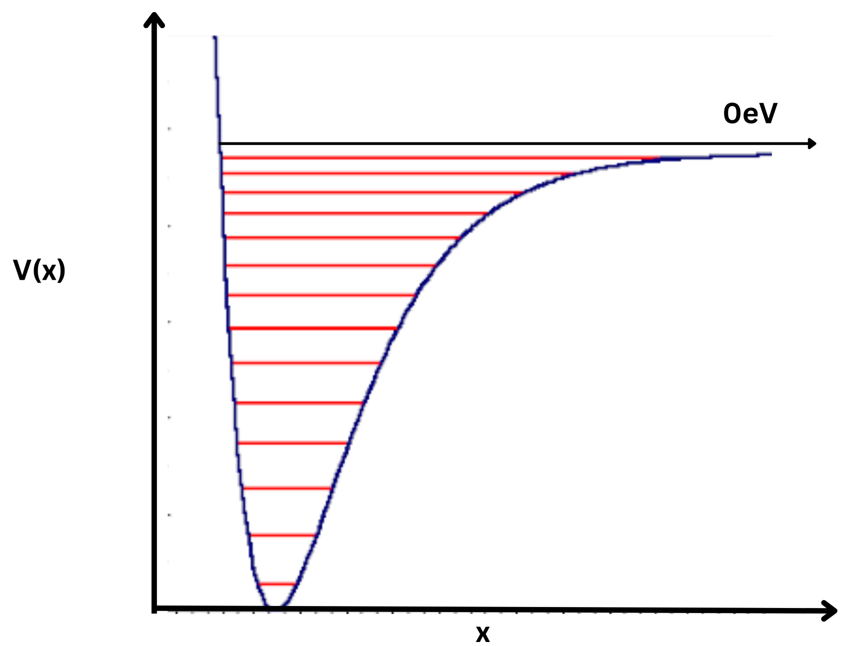

Figure 1: Potential Energy Graph for a Molecule

Question 3

In class, we solved the Schrodinger equation for a three-dimensional wavefunction. Using your knowledge from that, you can write down the wavefunctions for a 3-D infinite potential well. The well has infinite potential at all of its walls. The length, width, and height are L, L, and F respectively.

(a) Write down the wavefunction for the given potential well.

(b) Write down the energies for the ground state, 1st excited state, and the 2nd excited state.

(c) Are the energy levels degenerate for the ground state, 1st excited state, and 2nd excited state? Explain your answer.

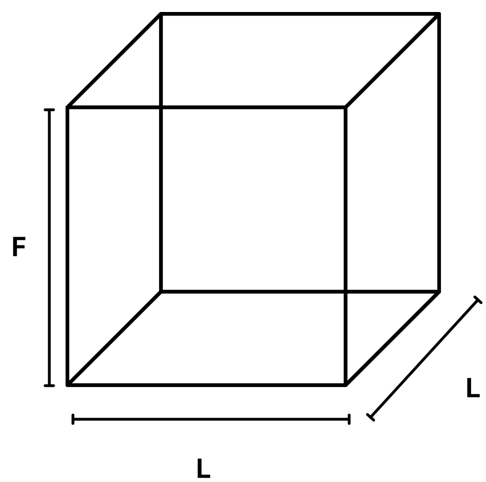

Figure 2: Infinite potential well

Question 4

What is the longest possible emitted wavelength for a harmonic oscillator with frequency ω?

Question 5

An electron with energy E, traveling from the left is faced by a potential energy barrier with energy V_O. The length of the barrier is w.

(a) Write down the wavefunction in regions I, II, and III, as shown in the Figure below.

(b) Find the probability that the electron tunnels through the potential barrier.

(c) Find the complete wavefunction for the given system using appropriate boundary conditions.

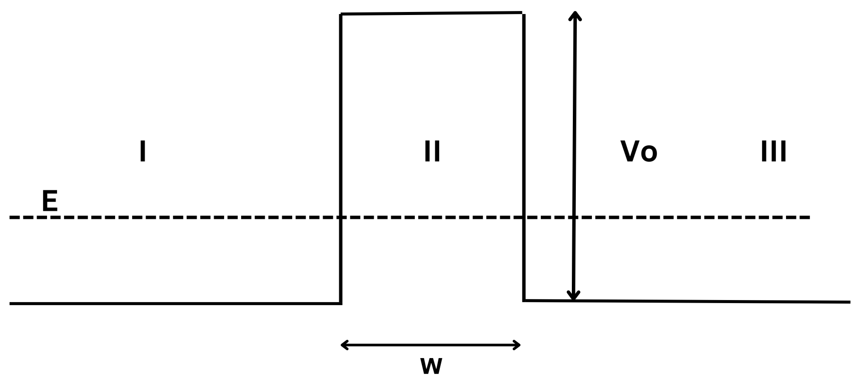

Figure 3: Potential barrier

Question 6

A proton's energy is 1.0MeV below the top of a 10-fm-wide energy barrier. What is the probability that the proton will tunnel through the barrier?

Question 7

Tennis balls travelling faster than 100mph routinely bounce off tennis rackets. At some sufficiently high speed, however, the ball will break through the strings and keep going. The racket is a potential-energy barrier whose height is the energy of the slowest string-breaking ball. Suppose that a 100g tennis ball travelling at 200mph is just sufficient to break the 2.0-mm-thick strings. Estimate the probability that a 120mph ball will tunnel through the racket without breaking the strings.

Question 8

For the potential step shown in the figure, the potential energy function is defined below

$$\begin{cases} 0 & x < 0 \\ V_0 & x \geq 0 \end{cases}$$

Find

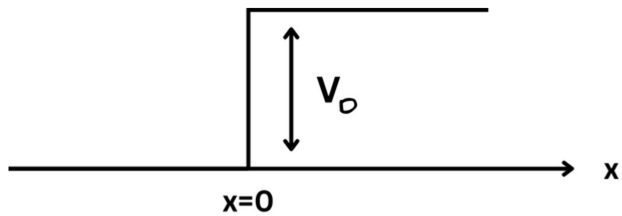

Figure 4: Potential Step for Question 8

(a) the wavefunction for a state with energy greater than V_0.

(b) the wavefunction for a state with energy, E, such that $0 < E < V_0$.

Question 9

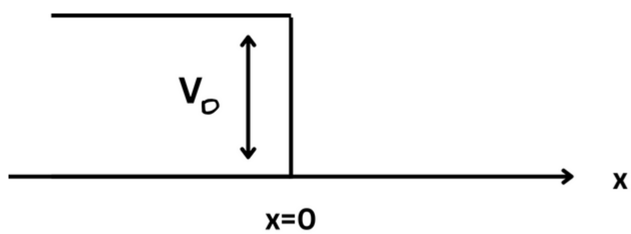

Figure 5: Potential Step for Question 9

For the potential step shown in the figure, the potential energy function is defined below

$$\begin{cases} 0 & x < 0 \\ -V_0 & x \geq 0 \end{cases}$$

Find

(a) the wavefunction for a state with energy greater than V_0.

(b) the wavefunction for a state with energy, E, such that $0 < E < V_0$.

Question 10

Consider a particle in an infinite potential well. Given that this particle is in the first excited state,

(a) Plot the probability distribution function for its position.

(b) Find the probability of finding the particle between $x = 0$ and $x = L$.

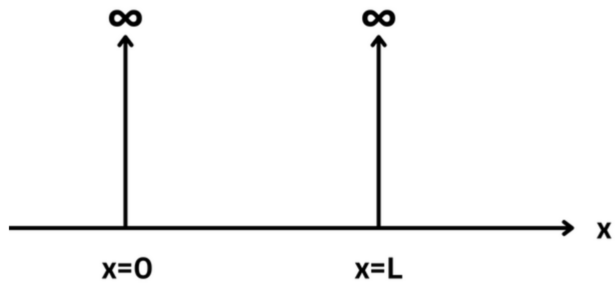

Figure 6: Infinite Potential Well for Question 10

(c) Find the probability of finding the particle between $x = 0$ and $x = \frac{L}{2}$.

(d) How much energy would it take for the particle to transition from the first excited state to the second excited state?

(e) Given that the aforementioned transition can be achieved by the absorption of a photon of wavelength 600 nm, find the type of photon and the length of the box.

Question 11

Consider the particle in the potential landscape shown. Sketch the wavefuntion for a state with energy, E, as shown on the diagram.

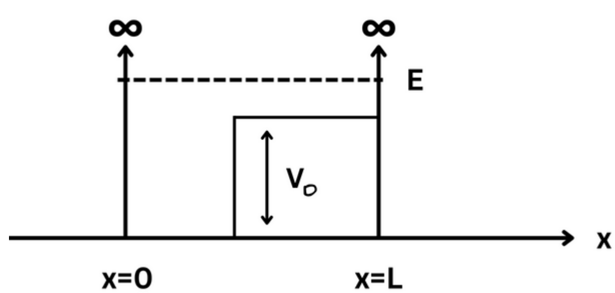

Figure 7: Potential Landscape for Question 11

This question does not require a full calculation- try to correctly represent the different amplitudes and wavelengths in different regions.

Question 12

Repeat Question 11 for the given potential landscape.

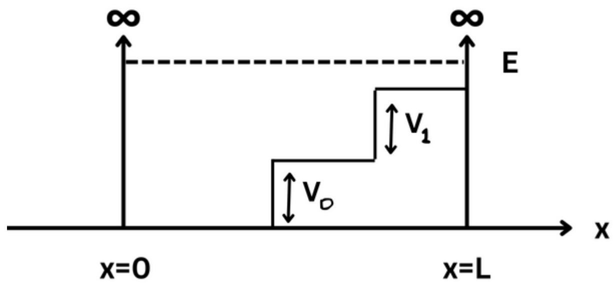

Figure 8: Potential Landscape for Question 12

Question 13

A particle's wavefuntion is given below.

$$\begin{cases} 0 & x < 0 \\ A\cos\left(\frac{2n\pi}{L}x\right) & 0 \le x \le L \\ 0 & x > L \end{cases}$$

Find A (in terms of L).