Assignment 1

1. (a) Classical mechanics provides us with a ratio between the angular momentum and magnetic moment due to a charged particle moving in a loop. This is often called the gyromagnetic ratio. Interestingly, such a ratio exists in quantum mechanics too. Calculate the gyromagnetic ratio, γ, for an electron, muon, proton, and neutron. Their Landé g-factors are $-2.002, -2.002, 5.58$, and -3.83, respectively.

   Now that we recognize that these particles have a magnetic moment, it is worth investigating the behavior of momenta placed inside a magnetic field. To this effect, find the Larmor frequency, ω, for an electron, muon, proton, and neutron placed inside a magnetic field, $B = 1$ T. Quote the masses you use in your calculations and comment on the relative values of frequency that you find.

   (b) We have been studying isolated magnetic moments. If one studies atoms or ions, one must bring to consideration both spin and orbital angular momentum of different particles.

   The Landé g-factor is a dimensionless quantity, calculated using quantum mechanics, that shows up inside the gyromagnetic ratio. Derive an expression for the Landé g-factor for one ion with orbital and spin quantum numbers, L and S. Refer to Appendix C. 9 from Blundell.

2. (a) Find an expression for magnetization in terms of the magnetic field and the temperature for a system of any general spin, j, and with concentration of spins, n. Check if this result boils down to the much simpler expression for the magnetization of an isolated electron with only spin angular momentum.

   (b) Find the susceptibility. What happens to the susceptibility in the high temperature limit? Does the Curie law hold?

   (c) Plot the susceptibility as a function of B/T for various values of j: $1/2, 1, 3/2, 10, \infty$. Be careful with the units. Now, comment on the plot in which j approaches infinity. Finally, explain if it is possible to ascertain from any one of these plots whether the system is paramagnetic or not.

3. The Landau-Lifshitz-Gilbert equation models the precession of a magnetic moment placed inside a magnetic field, and it also takes care of a damping affect that aligns the magnetic moment in the direction of the applied magnetic field:

$$\frac{d\boldsymbol{\mu}}{dt} = \gamma(\boldsymbol{\mu} \times \boldsymbol{B}) + \frac{\alpha}{\mu}\left(\boldsymbol{\mu} \times \frac{d\boldsymbol{\mu}}{dt}\right)$$

   Here α is a constant that characterizes the damping.

   (a) Show that the above equation can be written as,

$$\frac{d\boldsymbol{\mu}}{dt}\left(1+\alpha^2\right) = \gamma(\boldsymbol{\mu}\times\boldsymbol{B}) + \frac{\alpha\gamma}{\mu}(\boldsymbol{\mu}\times(\boldsymbol{\mu}\times\boldsymbol{B})). \tag{1}$$

(b) Write a computer program that simulates the damping of the moment for various choices of α and initial conditions on $\boldsymbol{\mu}$. Choose dimensionless units, B, $\gamma = 1$. For this simulation, a set of coupled differential equations must be solved.