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The Atwood machine, a traditional device in physics, has been an invaluable tool for demonstrating and understanding the fundamental principles of Newtonian mechanics, especially Newton's second law.[1] Some time ago, we published in this column a proposal that makes use of the smartphones and their built-in sensors to implement an updated version of the experiment but keeping its essence unchanged.[2] Now, we propose a *circular* Atwood machine designed to study the relationship between the external torque on a rigid disk and its angular acceleration mediated through the moment of inertia. In our proposal, quantitative measuments are achieved by means of a smartphone's angular velocity sensor. The moment of inertia measured was successfully contrasted with the value obtained from the period of oscillations of the device hanging from a point at its periphery.

The circular Atwood machines

Adapting the traditional Atwood machine to be able to perform experiments involving moment of inertia and angular momentum offers interesting opportunities. This modified setup involves a rotating disk influenced by controlled external torque, enabling precise manipulation of the moment of inertia and torque. The device facilitates straightforward measurement of angular acceleration, crucial for understanding the correlation between external torque and the disk's angular momentum, as depicted in Fig. 1.

The device is composed of a disk that can rotate around the vertical axis subject to an external torque given by a thread with tension T by means of a pulley of radius R. In general, we must take the rolling resistance in the bearings τ_{roz} into account. We denote as I the device's moment of inertia about a vertical axis through the center of mass, and we denote as α the angular acceleration. The total external torque is equal to the moment of inertia times the angular acceleration. Considering the vertical axis and that the total external torque is the sum of the torque exerted by the pulley minus the rolling friction, we can write

$$TR - \tau_{roz} = I\alpha. \tag{1}$$

On the other hand, the tension T is easily found as

$$m_0 g - T = m_0 a, \tag{2}$$

where a is the vertical acceleration of the mass and, since the string does not slip with respect to the pulley, can be related to the angular acceleration of the disk as

$$a = \alpha R. \tag{3}$$

The moment of inertia of the device can be varied by symmetrically placing the two masses shown in Fig. 1 at different distances r from the center of rotation. For simplicity, we write the moment of inertia as the sum of the device's moment of inertia I_D, which remains constant, and the (variable) contributions of the masses

$$I = I_D + 2mr^2. \tag{4}$$

From the previous equations, we can eliminate T and a to obtain

$$\frac{1}{\alpha} = \frac{I_D + 2mr^2 + m_0 R^2}{m_0 gR - \tau_{roz}}, \tag{5}$$

expressing a relationship between the inverse of the angular acceleration and the distance r. In principle, it is possible to design different experiments with the circular Atwood machine. In this article, we record the angular acceleration values as a function of r, i.e., placing the masses in the different slots of the support.

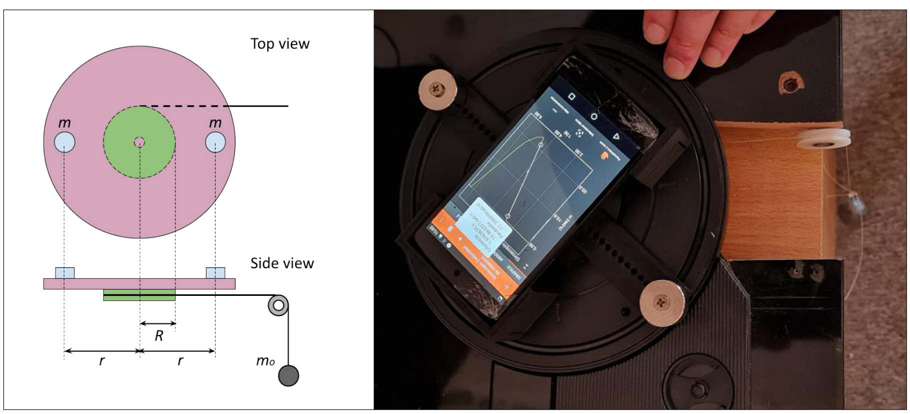

Fig. 1. Experimental setup: schematic representation (left panel) and photograph (right panel).

The experiment

The experimental device, shown in Fig. 1, was built from an old record player and includes a 3D-printed plastic part, masses, and a spring. The plastic piece is a support where the smartphone can be held, and has slots to place the masses, each $m = 0.0521(5)$ kg, and is attached to the center of the tray. The design was made in Tinkercad and is available for anyone interested to suit their own experiment.[3] The slots are equally spaced and located on a diameter so as to vary the distance to the center of rotation r in a regular way. The disk is connected

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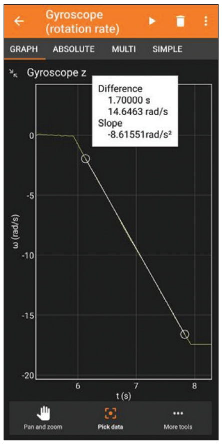

Fig. 2. Screenshot of the Phyphox app used to register angular velocity and the angular acceleration obtained by means of the slope of the graph.

to a hanging mass, $m_0 = 0.1265(5)$ kg, by means of pulley with radius $R = 0.0259(1)$ m. The weights of the strings and the pulley can be neglected.

A smartphone Nexus 5 is placed on the holder to register the angular velocity by means of the gyroscope (rotation sensor) included in the built-in MPU6515 MEMS chip. In this experiment, we use the Phyphox app[4] to record the angular velocity. For each location of the masses, we record the angular velocity, and as shown in the screenshot of Fig. 2, we identify the falling interval of the hanging mass and calculate the slope of the graph corresponding to the angular acceleration.

Figure 3 summarizes the experimental results. Taking into consideration Eq. (5), we plot the inverse of the angular acceleration as a function of r^2. We

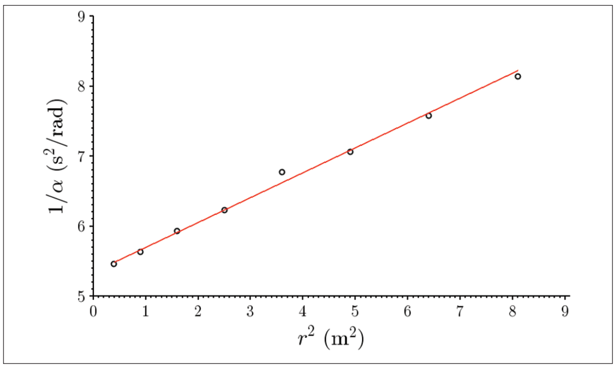

Fig. 3. Linearized relationship between the angular velocity and the location of the masses. Experimental results (crosses) and linear fit (red line).

performed a linear regression of the form $1/\alpha = ar^2 + b$, where

$$a = \frac{2m}{m_0 gR - \tau_{roz}} \quad \text{and} \quad b = \frac{I_D + m_0 R^2}{m_0 gR - \tau_{roz}}, \tag{6}$$

obtaining a linear correlation coefficient of $r > 0.99$. From these equations, we obtained $\tau_{roz} = 0.00232(2)$ N · m and $I_D = 1.48(2) \times 10^{-3}$ kg · m².

A direct measure of the moment of inertia

To obtain the moment of inertia, the device was attached to a point on the periphery and allowed to oscillate in a vertical plane as indicated in Fig. 4. To measure the period, we again used the rotation sensor with the help of the Phyphox app; in this case, the period obtained by averaging over 10 oscillations was $T = 0.735(2)$ s. The total mass of the device was $M_D = 0.3770(5)$ kg (including disk, pulley, support, and smartphone), and the distance from the point of oscillation to the center of rotation was $d = 0.0910(2)$ m. Applying the Steiner parallel-axis theorem to the period of oscillation of the system,

$$T = 2\pi\sqrt{\frac{I_D + M_D d^2}{M_D g d}}, \tag{7}$$

we find $I_D = 1.48(6) \times 10^{-3}$ kg · m², in great concordance with the value obtained from the angular acceleration fit.

Conclusion

We introduced an innovative approach using emerging technologies to transform the Atwood Machine into a tool for studying circular dynamics. This experiment can be set up with basic materials, including an old record player tray, a 3D-printed holder, bearings, and a smartphone with an angular velocity sensor. By measuring angular acceleration and linking it to the device's angular momentum, the experiment provides valuable insights. Adjusting parameters allows analysis of friction at the bearing and the device's moment of inertia, contributing to a deeper understanding of fundamental physics concepts.

Fig. 4. Direct measurement of the moment of inertia; the period of oscillation was registered using the Phyphox app (left picture) and averaging over 10 oscillations (right picture).

References

1. T. B. Greenslade Jr., "Atwood's machine," *Phys. Teach.* **23**, 24–28 (1985).
2. M. Monteiro, C. Stari, C. Cabeza, and A. C. Marti, "The Atwood machine revisited using smartphones," *Phys. Teach.* **53**, 373–374 (2015).
3. Editable layout to 3D print the support for circular Atwood in Tinkercad, https://bit.ly/3D-CircularAtwood.
4. S. Staacks, S. Hütz, H. Heinke, and C. Stampfer, "Advanced tools for smartphone-based experiments: phyphox," *Phys. Educ.* **53**, 045009 (2018).

iPhysicsLabs *are short articles featuring uses of smartphone technology in physics teaching. To submit, please email Jochen Kuhn (jochen.kuhn@lmu.de) and Patrik Vogt (vogt@ilf-mainz.de).*