Assignment 2

1. (a) A tiny magnetic moment $\vec{\mu}_1$, is placed at the origin, its magnetic vector potential at a postion $\vec{r}$ is

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{\mu}_1 \times \vec{r}}{|\vec{r}|^3} \tag{1}$$

Use $\vec{B} = \vec{\nabla} \times \vec{A}$ and a vector identity to verify that two magnetic dipoles $\vec{\mu}_1$ and $\vec{\mu}_2$ separated by $\vec{r}$ have a magnetic dipolar energy equal to

$$E = \frac{\mu_0}{4\pi r^3}\left[\vec{\mu}_1 \cdot \vec{\mu}_2 - \frac{3}{r^2}\left(\mu_1 \cdot \vec{r}\right)\left(\vec{\mu}_2 \cdot \vec{r}\right)\right] \tag{2}$$

(b) Calculate the dipolar energy between two protons separated at $1\mathring{A}$ and $10\mathring{A}$ when their spin are (i) parallel (ii) antiparallel.

2. We now turn our attention to the quantum mechanical consequences of indistinguishable particles: exchange interaction. For example, one can show that the average distance between two distinguishable and indistinguishable particles is different (you may look at the chapter on exchange forces in Griffith's *Introduction to Quantum Mechanics*). In class, we looked at another consequence of indistinguishable particles in quantum mechanics, the exchange integral.

(a) We consider the Helium atom, and your task is to follow steps from class to derive (i) zeroth order term and first order correction to the ground state energy of the electrons (ii) zeroth order term and first order correction for the 1st excited state of the electrons. Remember that for (ii) there are more possibilities of wave functions (as we saw in class), and you need to calculate (ii) for all possible wave functions. Comment on the splitting of energy of the singlet and the triplet state.

Hint: Consider taking a look at the chapter on identical particles in Townsend's *Modern Approach to Quantum Mechanics.*

3. In class, we looked at a model of the H_2^+ ion (two protons and one electron). We begin here because we are interested in later studying the Hydrogen molecule (two protons and two electrons). Let us try to first fully grapple with the aforementioned ion.

(a) Begin with writing down the Hamiltonian of the H_2^+ ion. Explain the meaning of each term and draw a figure that shows the coordinates chosen.

(b) Choose a basis to write down the Hamiltonian. You must explain the choice of basis. Also write down the spatial overlap of the basis.

(c) Write down the Hamiltonian in the chosen basis. Find the eigenstates (in terms of your basis, of course) and eigenvalues of the Hamiltonian. Be warned that this part requires calculations quite detailed and several integrals. Hint: Refer to class notes, Townsend's *Modern Approach to Quantum Mechanics*, and Griffith's *Introduction to Quantum Mechanics*.

(d) Call these eigenstates, the bonding and anti bonding states: $|\phi_b\rangle$ and $|\phi_a\rangle$. Find the spatial overlap of these states, $\langle\vec{r}|\phi_b\rangle$ and $\langle\vec{r}|\phi_a\rangle$. Plot both of these wave functions.