Assignment 5

1. Suppose E_F is the Fermi energy (in $3D$) for electrons in a solid. Given the usual definition of x and x_o, the integrated density of states are

$$\widetilde{G}_{3D}(x) = \frac{2}{3}\hbar\omega_c \frac{G_{3D}(x_o)}{\sqrt{x_o}} x^{3/2}, \tag{1}$$

$$\widetilde{G}_{3D}(x) = \hbar\omega_c \frac{G_{3D}(x_o)}{\sqrt{x_o}} \sum_{l=0}^{l_{\max}} \sqrt{x-(l+1/2)}\ \Theta(x-(l+1/2)), \tag{2}$$

where $\Theta(x-(l+1/2))$ is the Heaviside step function.

(a) Derive an expression for

$$U^{(B)} = \int_0^{E_F+\delta E_F} dE\, G_{3D}(x)\, E.$$

(b) Derive expression (2) above from $G_{3D}(x)$.

(c) Derive an expression for $\frac{n^{(B)}}{n^{(0)}}$, where $n^{(B)}$ is the number of states that exist uptil the energy E_F with the magnetic field turned on and $n^{(0)}$ is the number density in the absence of a magnetic field. Now, plot $\frac{n^{(B)}}{n^{(0)}}$ and interpret the result.

2. In class we plotted δx (change in Fermi energy with the application of a magnetic field) with respect to x_o. Your task is to plot this change with respect to B instead of x_o. Identify the region corresponding to high magnetic fields when only the lowest Landau levels (say, $l = 0, 1, 2, 3$) are occupied. At what values of x_o, would we get the magnetic ground state ($l = 0$)?

3. (a) Write down the Hamiltonian for free electrons in the Landau Gauge: $\vec{A} = (0, Bx, 0)$. Note that in class we used $\vec{A} = (-By, 0, 0)$.

(b) What are the eigenstates of this Hamiltonian? Do these eigenstates depend on the choice of gauge?

(c) Express this Hamiltonian in a form similar to a harmonic oscillator.

4. This is very, very simple question. Write down the expression for magnetic susceptibility for:

(a) paramagnetic ions for spin-1/2 in the high temperature limit,

(b) a free electron gas (Pauli paramagnetism),

(c) the Landau magnetization term.

5. (a) A $2D$ solid has N free electrons. If all of these are to be in the $l = 0$ level (magnetic quantum limit), what should be the minimum magnetic field $B^{(\text{all in } l=0)}$ be? Keep in mind the degeneracy of the Landau levels.

   (b) What should the maximum field, $B^{(l_{\max})}$, be if all levels up to and including $l_{\max}$ are equally populated?

   (c) If $B^{(l_{\max}+1)} < B < B^{(l_{\max})}$, what is the total energy of the electrons?