Undamped, on - resonance

```
In[•]:= eqns = {u'[t] == 0, v'[t] == Ωr * w[t], w'[t] == -Ωr * v[t], w[0] == -1, v[0] == 0, u[0] == 0}
Out[•]= {u'[t] == 0, v'[t] == Ωr w[t], w'[t] == -Ωr v[t], w[0] == -1, v[0] == 0, u[0] == 0}

In[•]:= DSolve[eqns, {u, v, w}, t]
Out[•]= {{u → Function[{t}, 0], v → Function[{t}, -Sin[t Ωr]], w → Function[{t}, -Cos[t Ωr]]}}
```

Undamped, off - resonance

```
In[•]:= eqnsoffres = {u'[t] == δ * v[t], v'[t] == Ωr * w[t] - δ * u[t],
        w'[t] == -Ωr * v[t], w[0] == -1, v[0] == 0, u[0] == 0}
Out[•]= {u'[t] == δ v[t], v'[t] == -δ u[t] + Ωr w[t], w'[t] == -Ωr v[t], w[0] == -1, v[0] == 0, u[0] == 0}

In[•]:= DSolve[eqnsoffres, {u, v, w}, t]
```

$$\text{Out[•]= } \left\{\left\{u \to \text{Function}\left[\{t\}, \frac{e^{-t\sqrt{-\delta^2-\Omega r^2}}\left(-1+e^{t\sqrt{-\delta^2-\Omega r^2}}\right)^2 \delta\,\Omega r}{2\left(\delta^2+\Omega r^2\right)}\right],\right.\right.$$

$$v \to \text{Function}\left[\{t\}, \frac{e^{-t\sqrt{-\delta^2-\Omega r^2}}\left(-1+e^{2t\sqrt{-\delta^2-\Omega r^2}}\right)\Omega r\sqrt{-\delta^2-\Omega r^2}}{2\left(\delta^2+\Omega r^2\right)}\right],$$

$$\left.\left.w \to \text{Function}\left[\{t\}, -\frac{e^{-t\sqrt{-\delta^2-\Omega r^2}}\left(2e^{t\sqrt{-\delta^2-\Omega r^2}}\,\delta^2+\Omega r^2+e^{2t\sqrt{-\delta^2-\Omega r^2}}\,\Omega r^2\right)}{2\left(\delta^2+\Omega r^2\right)}\right]\right\}\right\}$$

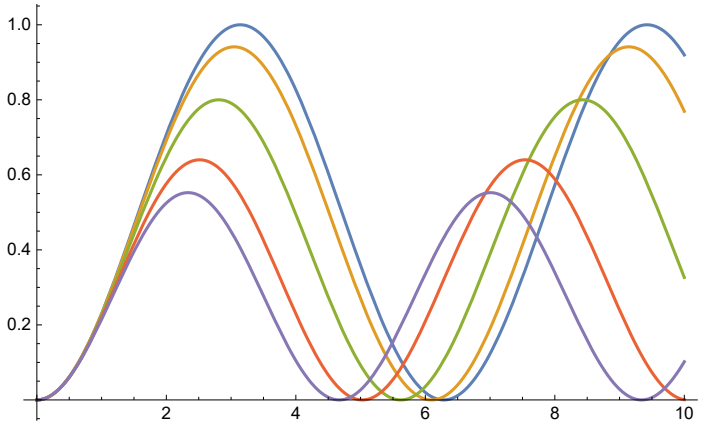
```
In[•]:= q = FullSimplify[FullSimplify[-(e^(-t Sqrt[-δ^2-Ωr^2]) (2 e^(t Sqrt[-δ^2-Ωr^2]) δ^2 + Ωr^2 + e^(2 t Sqrt[-δ^2-Ωr^2]) Ωr^2)) / (2 (δ^2 + Ωr^2))] + 1) / 2]
```

$$\text{Out[•]= } -\frac{\Omega r^2\,\text{Sinh}\left[\frac{1}{2}t\sqrt{-\delta^2-\Omega r^2}\right]^2}{\delta^2+\Omega r^2}$$

```
In[•]:= zz = FullSimplify[q /. {δ → 0.5, Ωr → 1}]
Out[•]= 0.8 Sin[0.559017 t]^2
```

In[•]:=
```
Plot[{q /. {δ → 0, Ωr → 1}, q /. {δ → 0.25, Ωr → 1},
  q /. {δ → 0.5, Ωr → 1}, q /. {δ → 0.75, Ωr → 1}, q /. {δ → 0.9, Ωr → 1}}, {t, 0, 10}]
```

Out[•]=



Damped, on - resonance

In[1]:=
```
eqnsdamponres = {u'[t] == 0 - γ * u[t] / 2, v'[t] == Ωr * w[t] - γ * v[t] / 2,
  w'[t] == -Ωr * v[t] - γ * (w[t] + 1), w[0] == -1, v[0] == 0, u[0] == 0}
```

Out[1]=
$$\left\{u'[t] == -\frac{1}{2}\gamma\, u[t],\ v'[t] == -\frac{1}{2}\gamma\, v[t] + \Omega r\, w[t],\ w'[t] == -\Omega r\, v[t] - \gamma\,(1 + w[t]),\ w[0] == -1,\ v[0] == 0,\ u[0] == 0\right\}$$

In[3]:=
```
sol = DSolve[eqnsdamponres, {u, v, w}, t];
```

In[5]:=
```
Dimensions[sol]
```

Out[5]=
```
{1, 3}
```

In[14]:=
```
FullSimplify[(1 + FullSimplify[w[t] /. sol /. {γ → 0, Ωr → 1}]) / 2]
```

Out[14]=
$$\left\{\text{Sin}\left[\frac{t}{2}\right]^2\right\}$$

In[15]:=
```
pp = FullSimplify[(1 + FullSimplify[w[t] /. sol /. {Ωr → 1}]) / 2]
```

Out[15]=
$$\left\{\frac{1 + e^{-\frac{3\,t\,\gamma}{4}}\left(-\text{Cosh}\left[\frac{1}{4}\,t\,\sqrt{-16+\gamma^2}\,\right] - \frac{3\,\gamma\,\text{Sinh}\left[\frac{1}{4}\,t\,\sqrt{-16+\gamma^2}\,\right]}{\sqrt{-16+\gamma^2}}\right)}{2+\gamma^2}\right\}$$

In[22]:=
```
Plot[{pp /. γ → 0, pp /. γ → 0.2, pp /. γ → .4, pp /. γ → 1}, {t, 0, 30}]
```

Out[22]=