Quantum Optics: HW2 by Muhammad Sabieh Anwar

Please submit before 10 am, Thursday, 6 March 2025; only by the channel communicated

1. Any two level atom state can be expressed as a density matrix $\tilde{\rho}$. This can be expressed as a combination of Pauli matrices

$$\tilde{\rho} = \begin{pmatrix} \tilde{\rho}_{aa} & \tilde{\rho}_{ab} \\ \tilde{\rho}_{ba} & \tilde{\rho}_{bb} \end{pmatrix} = \frac{\mathbf{1}}{2} + a\hat{\sigma}_x + b\hat{\sigma}_y + c\hat{\sigma}_z, \tag{1}$$

where a, b and c are real numbers and can be considered to be components of a Bloch vector. Express these numbers in terms of u, v and w which are the real parameters defined in class in the context of optical Bloch equations. Hence, express $\tilde{\rho}$ in terms of u, v and w.

2. The optical Bloch equations are:

$$\frac{d}{dt}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = -\begin{pmatrix} \Omega_R \\ 0 \\ \delta \end{pmatrix} \times \begin{pmatrix} u \\ v \\ w \end{pmatrix} - \gamma \begin{pmatrix} u \\ v \\ 2(w+1) \end{pmatrix}. \tag{2}$$

Note that I am using δ (instead of $\delta\omega$) to represent the detuning to avoid confusion with the inversion parameter w.

   (a) Show that on-resonance and in the absence of damping, $\rho_{bb}(t) = \sin^2(\Omega_R t/2)$. Show all your calculations provided one starts in the ground state.

   (b) If we are off-resonance, and neither is there damping, nor an electric field applied, how will the Bloch vector evolve as a function of time? Assume the initial state $u = 1$ and $v = w = 0$. Explain your result physically.

3. Let's now come to the coupled differential equations of the probability amplitudes in the RWA:

$$\begin{aligned} \frac{dc_a}{dt} &= i\frac{\Omega_R}{2}\, e^{i(\delta\omega)t} c_b(t) && (3) \\ \frac{dc_b}{dt} &= i\frac{\Omega_R}{2}\, e^{-i(\delta\omega)t} c_a(t). && (4) \end{aligned}$$

When the applied radiation is on-resonance, derive an expression for the coherence $\rho_{ab}(t)$. Begin by construction a differential equation for ρ_{ab}. [Your answer should be $\rho_{ab}(t) = -i/2\, \sin(\Omega_R t)$.]

4. We are considering the transition between H's $3p_z$ and $2s$ orbitals:

$$\begin{aligned} \langle \mathbf{r}|3p_z\rangle &= \frac{\sqrt{2}}{81\sqrt{\pi}a_o^{5/2}}\,(6 - r/a_o)\, r\, \cos\theta\, e^{-r/(3a_o)} && (5) \\ \langle \mathbf{r}|2s\rangle &= \frac{1}{4\sqrt{2\pi}a_o^{3/2}}\,(2 - r/a_o)\, e^{-r/(2a_o)}, && (6) \end{aligned}$$

where a_o is the Bohr radius. The $3p_z$ orbital indicates that a z axis has been defined through some stimulus.

(a) Incident radiation is polarized along z axis and shines on the atom. Find the dipole moment matrix element between the two states. Quote your answer in Cm^2. I expect you to write the integral but solve it using a computer-based integrator.

(b) Estimate the Einstein coefficients B and A and hence the lifetime of the excited state. Furthermore, estimate the spontaneous emission decay rate A.

(c) Suppose the applied radiation is of constant amplitude E_0. We like the Rabi frequency to be at least as big as damping, i.e.

$\Omega_R = \gamma$. What E_0 (in Volts m^{-1}) is required to achieve this Rabi frequency? What intensity is needed (in $\mathrm{W\,m^{-2}}$)? Note that $I = (1/2)\, c\,\varepsilon |E_o|^2$.

(d) With such an electric field and a constant amplitude, how long will a $\pi/2$ pulse be?

(e) For this combination of Ω_R and γ (with no detuning), what are the values of S_0 (resonant saturation factor), w_∞ and $\rho_{bb}(\infty)$?

(f) Numerically solve optical Bloch equations and plot the excited state population for the condition $\Omega_R = \gamma$. Does saturation approach the value derived in part (e)? Assume no detuning.

(g) The experimenter thinks that the time in part (d) is too long. She therefore replaces this by a short Gaussian pulse of width $\tau = 10$ ns whose amplitude varies as,

$$E_o\, e^{-t^2/(2\tau^2)}. \tag{7}$$

What should be the peak amplitude of this $\pi/2$ pulse, E_o, be and how does it compare with the electric field amplitude determined in part (c)? What is the *peak* intensity?

5. Attempt Q9.6 from Fox's *Quantum Optics*.