

Phenomenon# 12

مشاہدہ # ۱۲

The Song of Fluids (Solution Manual)

سیالات کی سرگوشیاں (حل نامہ)

Aleeza Ayaz and Muhammad Sabieh Anwar

August 15, 2025

1a) Ink in Water: Diffusion

[Ans 1a.] The negative sign means that particles move from areas of high concentration to low concentration. This follows the natural way things spread out. If there are more particles in one place, they will move toward the area with fewer particles. The negative sign makes sure the math matches this real-world behavior. Without it, the equation would give results that go against what we observe, like suggesting that ink in water would clump together instead of spreading out.

[Ans 1b.] To find the units of the diffusion coefficient D , we first determine the units of the other terms.

Flux (J) represents the amount of substance passing through a unit area per unit time. Since the amount is measured in moles (mol), area in square meters (m^2), and time in seconds (s), the units of flux are:

$$J = \frac{\text{mol}}{m^2 \cdot s}$$

Concentration (C) is the amount of substance per unit volume:

$$C = \frac{\text{mol}}{m^3}$$

Since dC/dx represents how concentration changes with distance (m), its units are:

$$\frac{dC}{dx} = \frac{\text{mol}}{m^4}$$

Rearranging Fick's Law for D :

$$D = \frac{J}{dC/dx}$$

Substituting the units:

$$D = \frac{\frac{\text{mol}}{m^2 \cdot s}}{\frac{\text{mol}}{m^4}}$$

Canceling mol:

$$D = \frac{m^2}{s}$$

The units of D (m^2/s) tell us that diffusion describes how fast something spreads out over an area as time passes. A larger D means the substance spreads faster, while a smaller D means it spreads slower.

[Ans 1c.] The rate of diffusion depends on:

1. Molecular size: Larger molecules move more slowly.
2. Viscosity of the medium: A denser medium slows diffusion.
3. Temperature: Higher temperatures increase molecular movement, enhancing diffusion.
4. Interaction with the medium: Molecules that interact strongly with the medium (e.g., charged particles in water) may diffuse more slowly.

These factors appear in Fick's Second Law, which considers how concentration changes over time and includes the *diffusion coefficient* D , which encapsulates these hidden properties.

[Ans 1d.] The two graphs do not look the same. r vs. t is curved while r^2 vs. t is a straight line graph.

[Ans 1e.] The slope of r vs. t graph decreases, thus it does not grow steadily. This shows us that as concentration gradient decreases, the rate of diffusion also decreases. This follows from Fick's Second Law, which governs how diffusion spreads over time.

[Ans 1f.] The relationship between r^2 vs. t is a linear one. However, the graph might not look exactly the same at every point due to experimental errors, variations in conditions (like temperature), or the initial drop settling due to gravity, which might not show true diffusion. At very short times, the ink may be affected by gravity before pure diffusion takes over, making the graph look different at the beginning. ***Students may also suggest other appropriate reasons based on their system, which could further explain any deviations in the graph.***

[Ans 1g.] We already know that from our graph that r^2 vs. t is a linear relationship, so:

$$r^2 = \text{constant} \times t$$

To find the units of this constant, recall that the units of r^2 are m^2 and the units of t are s . Since both sides must have the same units, the constant must have units of m^2/s , which are the same as the diffusion coefficient D .

However, the constant could contain a numerical factor. These factors do not always have units—sometimes they are just numerical values that adjust the relationship. ***This is something to be mindful of when deriving equations.***

For our specific case, the relationship is known to be:

$$r^2 = 4Dt$$

This factor of 4 comes from the solution of the diffusion equation, which is derived in the appendix of the main manual.

[Ans 1h.] To estimate the slope, plot your values in Excel, MATLAB, or Python (whichever is more comfortable for you) and find the line of best fit. From that, you'll get an approximate value of D . After comparing your result with typical values of D for diffusion in water, you can try to figure out what caused the deviation. ***Again, students are encouraged to find answers based on their specific systems.***

[Ans 1i.] Diffusion never truly stops, but it slows down as concentration differences get smaller. Over time, diffusion reaches a point called ***equilibrium***, where molecules are still moving, but there is no overall change in concentration. This is because diffusion happens due to random motion, which never completely stops.

Here are the sample plots:

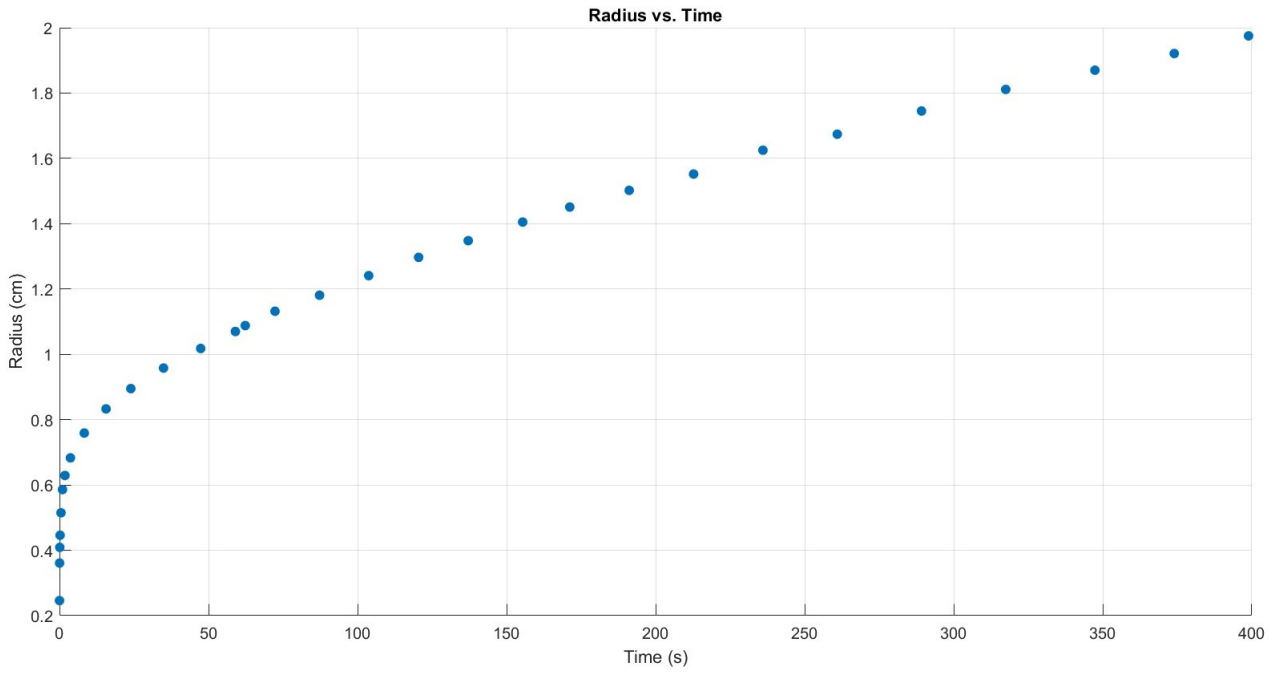


Figure 1: r vs. t graph from experimental data (1)

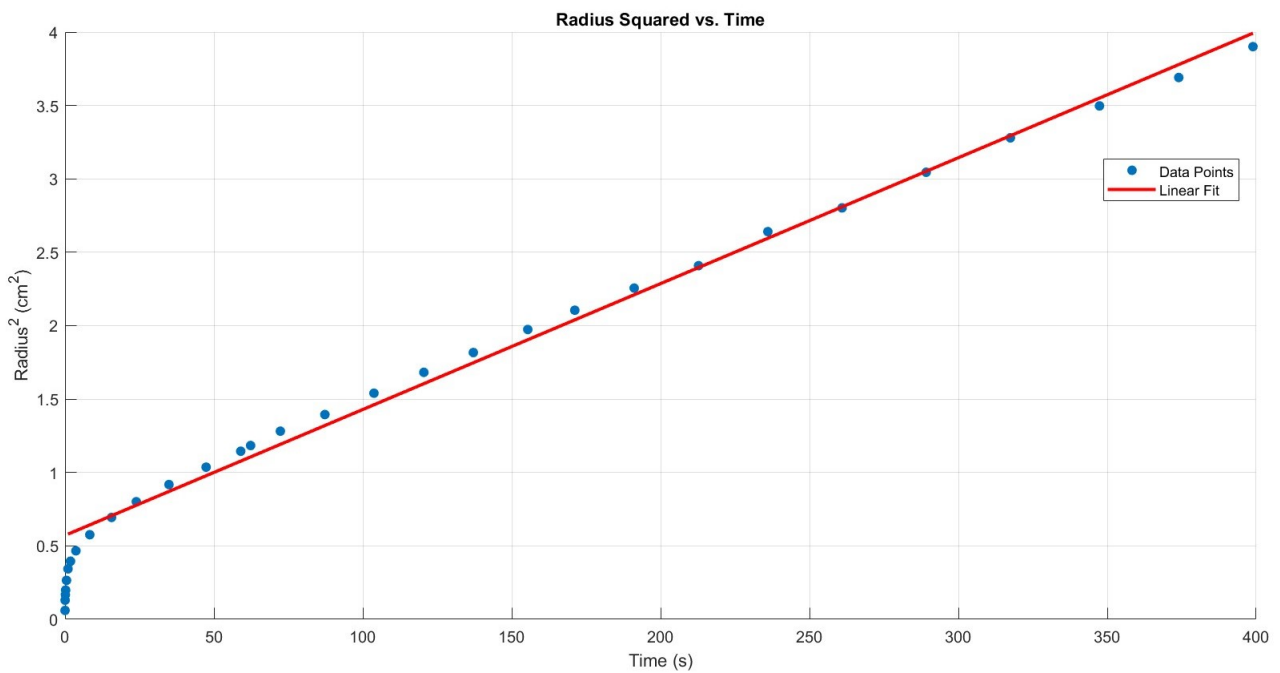


Figure 2: r^2 vs. t graph from experimental data (1), Linear Fit Equation: $y = 0.0086x + 0.5717$, R-squared Value: 0.9933.

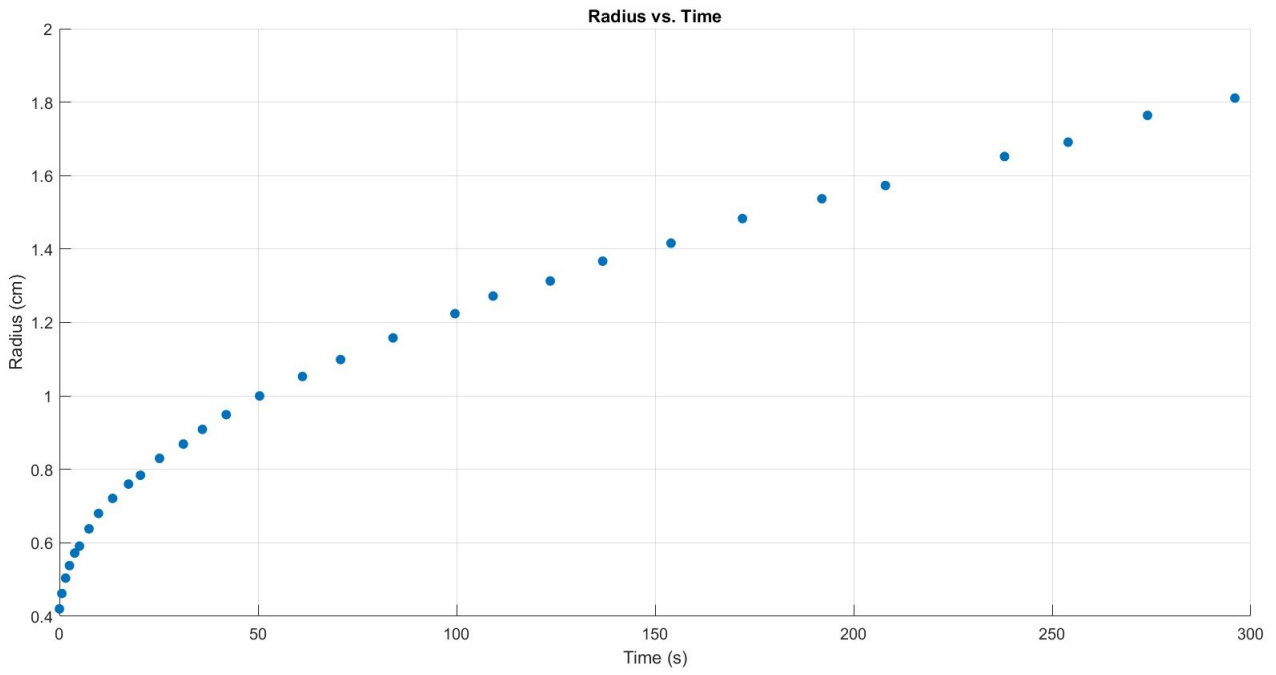


Figure 3: r vs. t graph from experimental data (2)

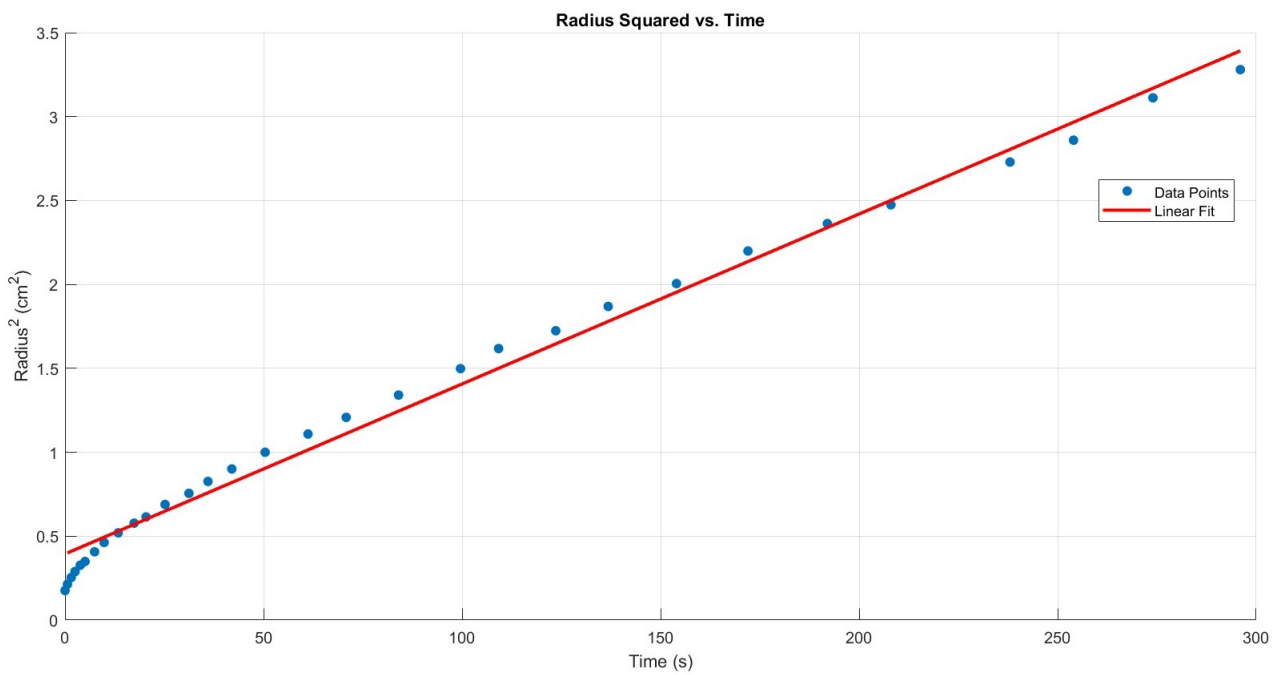


Figure 4: r^2 vs. t graph from experimental data (2), Linear Fit Equation: $y = 0.0101x + 0.3942$, R-squared value: 0.9910.

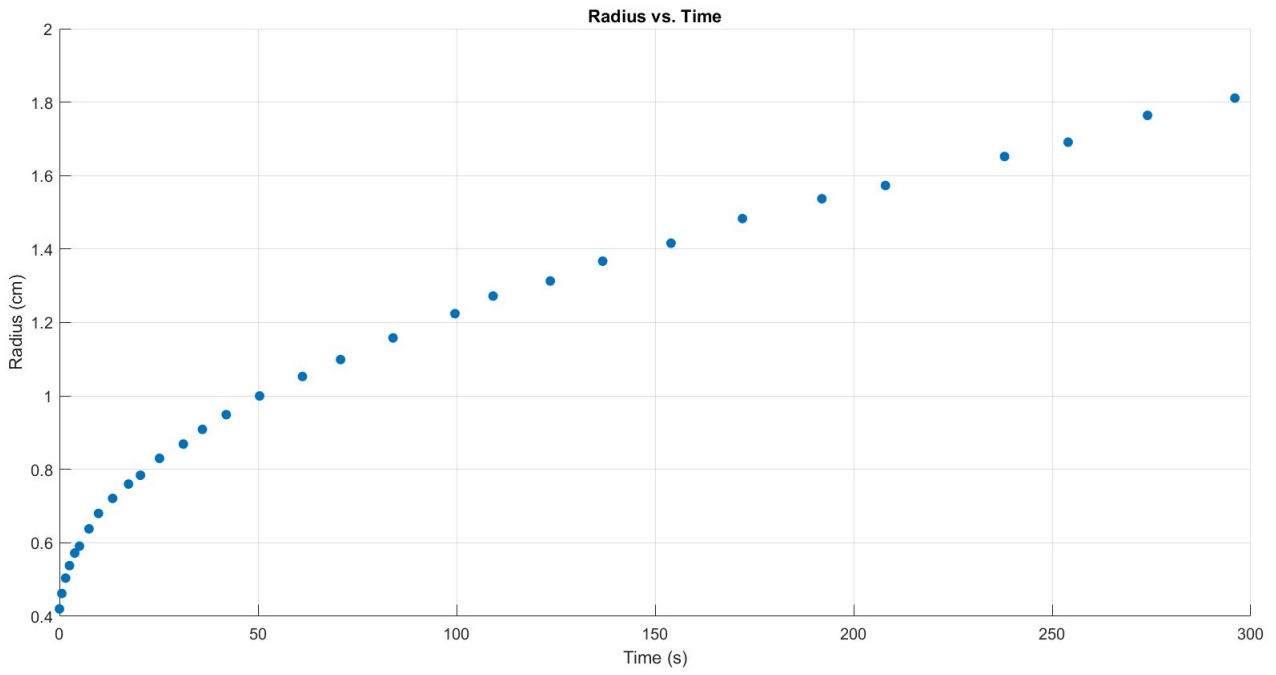


Figure 5: r vs. t graph plotted from experimental data (2).

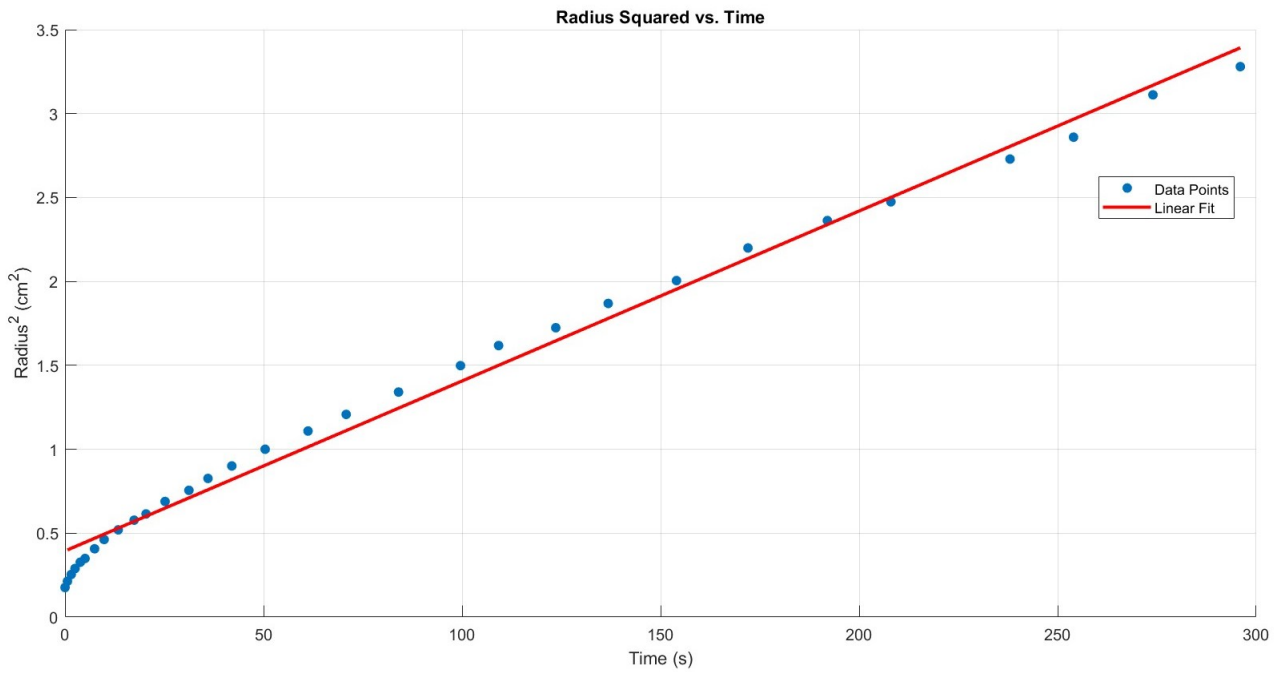


Figure 6: r^2 vs. t graph from experimental data (2), Linear Fit Equation: $y = 0.0101x + 0.3942$, R-squared value: 0.9910.

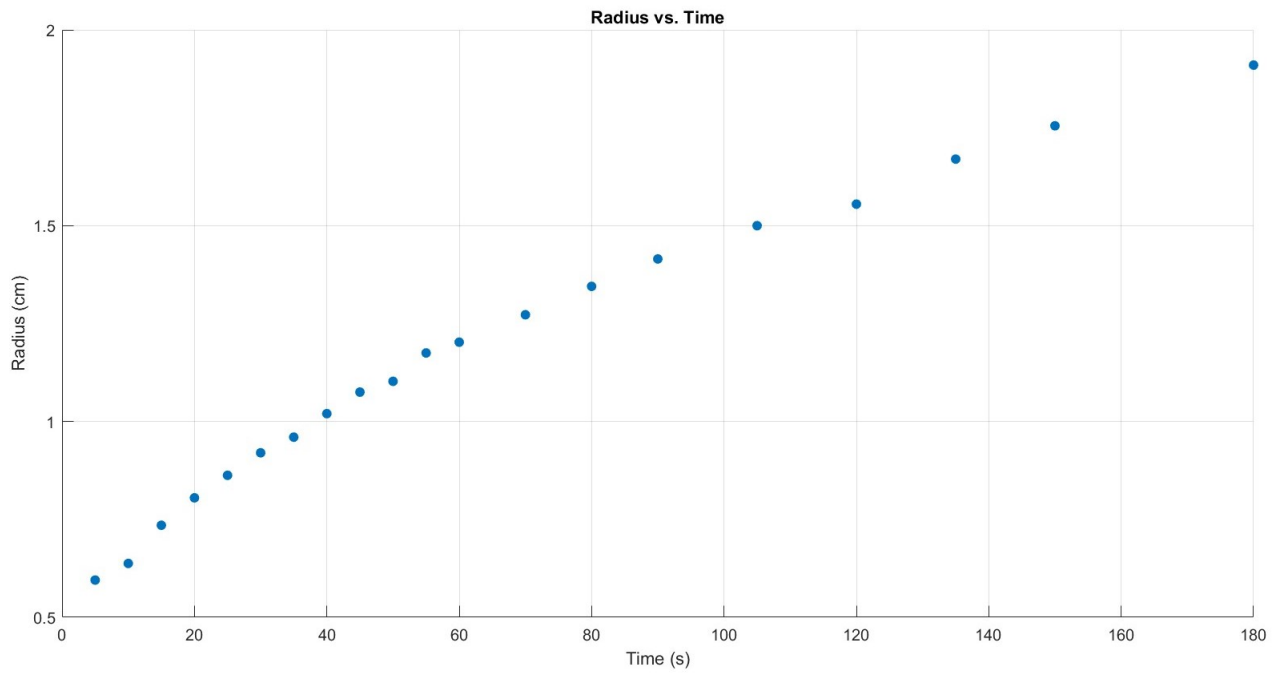


Figure 7: r vs. t graph plotted using data from [1].

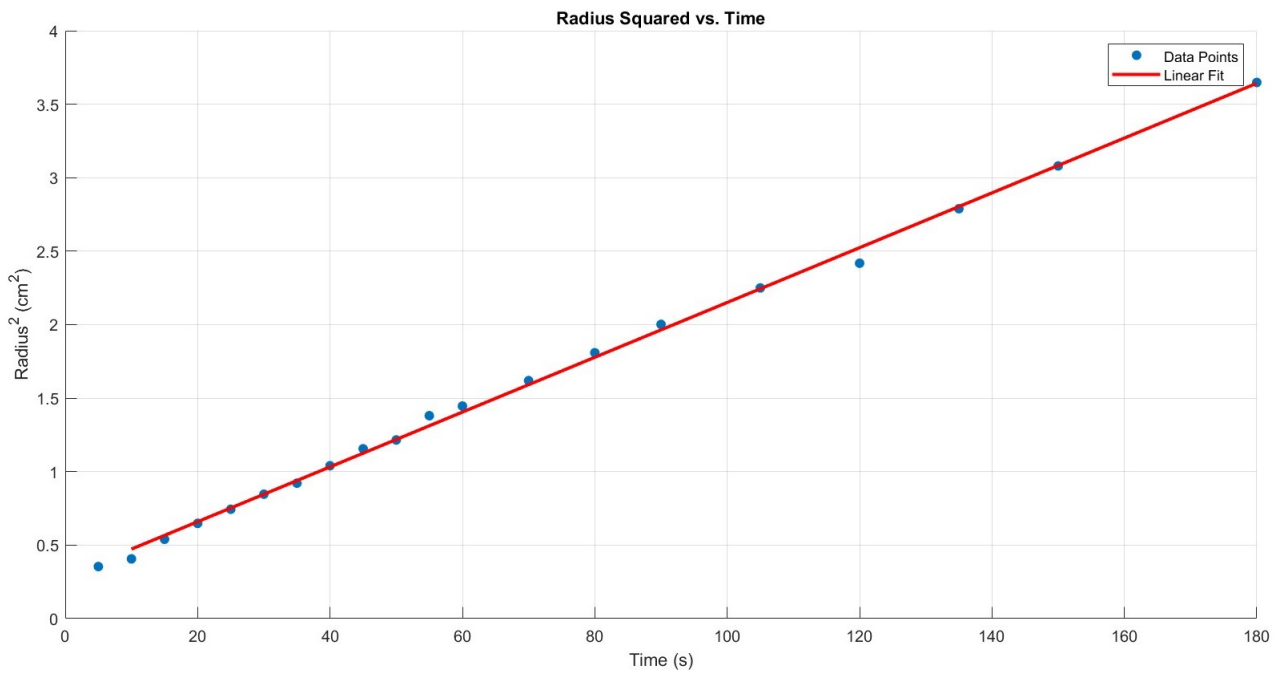


Figure 8: r^2 vs. t graph plotted using data from [1], Linear Fit Equation: $y = 0.0186x + 0.2865$, R-squared value: 0.9982.

1b) Ink in Water: Marangoni Flows

[Ans 1j.] *Students will make predictions.*

[Ans 1k.] The drop should spread rapidly.

[Ans 1l.] The drop should start to contract after rapidly spreading. This is because the alcohol in the ink will start to evaporate creating Marangoni flow in the opposite direction.

[Ans 1m.] Students should observe that when pure alcohol is added, it spreads faster and more chaotically compared to ink.

[Ans 1n.] Compared to ink, alcohol's spread is more irregular and less controlled. Ink moves outward in a more structured way, while alcohol disperses rapidly, often forming swirling or unpredictable patterns. Additionally, alcohol evaporates much faster than ink, which can further affect its spread.

[Ans 1o.] Since alcohol has very low surface tension and high volatility, it spreads rapidly and evaporates quickly, leading to short-lived, chaotic motion, unlike ink, which remains visible and continues spreading more gradually compared to pure alcohol.

Further reading is suggested for students interested in this phenomenon. Since science is always evolving, new findings may challenge previous ones. This is a relatively less explored phenomenon, so students are encouraged to investigate, make predictions, and question existing explanations. In addition to the listed references, they are encouraged to search for relevant materials on their own.

2) Marangoni Bursting

[Ans 2a.] Threshold IPA mass fraction = 0.35 ± 0.02 , as per the original article [2]. Below this value, the drop remains stable on the oil bath and gradually evaporates without fragmenting.

[Ans 2b.]

1. $r_m(t)$ initially increases as the droplet spreads, reaching a peak value $r_{max}(t)$, before decreasing as fragmentation occurs.
2. $r_o(t)$ increases as new droplets are ejected outward from the central droplet. However, we do observe contraction and coagulation after on.

[Ans 2c.] Normalization is useful because it removes dependencies on specific initial conditions, such as droplet size, allowing trends to be compared across different experiments. It also helps identify universal scaling behaviors by collapsing data onto a single curve. Since different trials may have varying absolute values for r_m , r_o and t , normalization allows us to compare patterns rather than focusing on the raw numbers, making it easier to analyze the overall behavior of the system.

[Ans 2d.] When we normalized our values, the graphs for r_m vs. t and r_o vs. t collapsed onto a single curve, showing that the system follows a self-similar behavior. This means that, despite differences in initial conditions, the spreading and breakup process is governed by the same fundamental physics. Normalization helps us compare different datasets more easily by removing the influence of initial size or concentration. *Students are highly encouraged to read the original article for a deeper understanding of this phenomenon and its implications.*

[Ans 2e.] As the mass fraction increases, the number of "fingers" also increases, leading to more ejected droplets. At higher mass fractions, the fragmentation occurs more rapidly and at a finer scale, making it increasingly difficult to observe individual breakup events.

Here are the sample results:

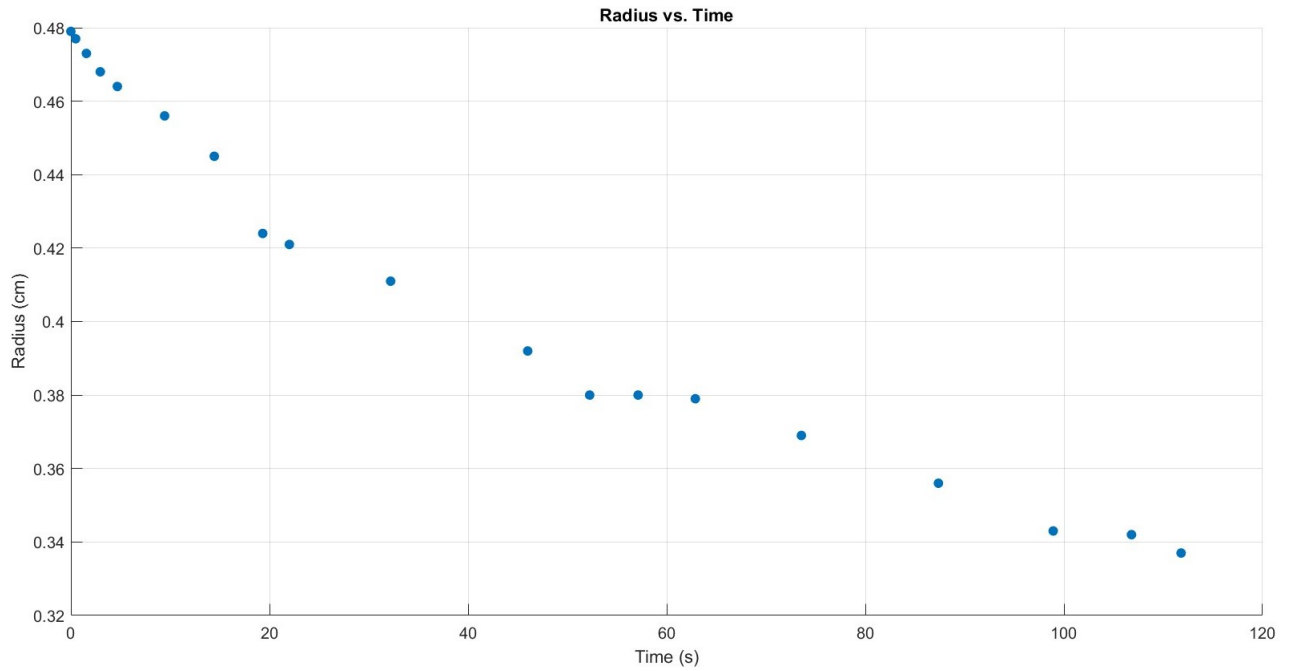


Figure 9: r vs. t graph for 0.30 mass fraction of IPA (less than threshold mass fraction).

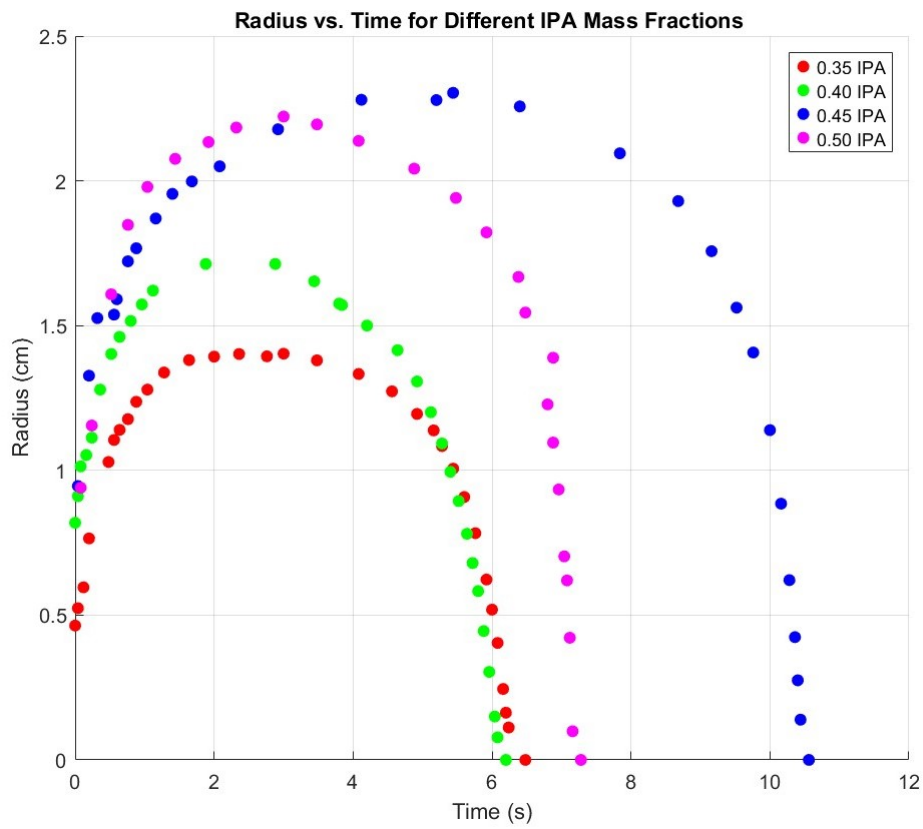


Figure 10: r_m vs. t graph for various IPA mass fractions

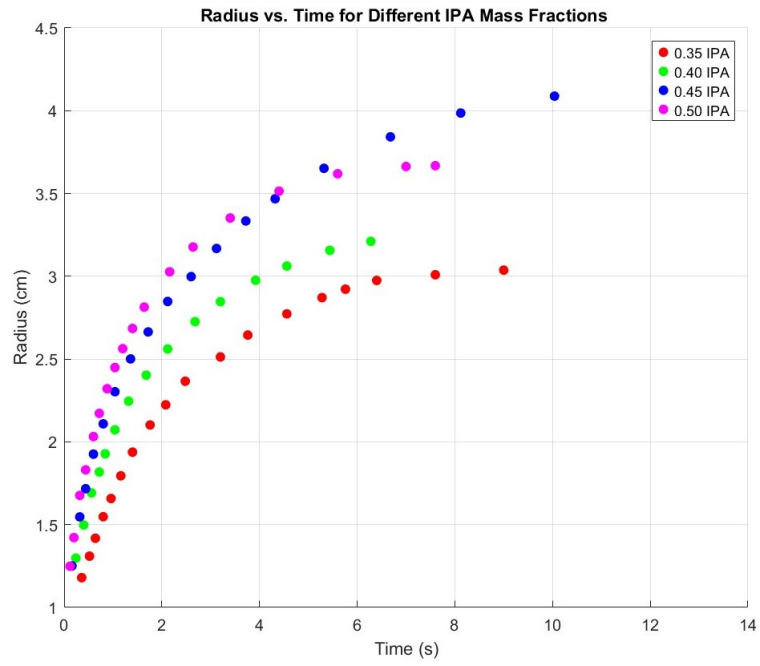


Figure 11: r_o vs. t graph for 0.30 mass fraction of IPA (less than threshold mass fraction).

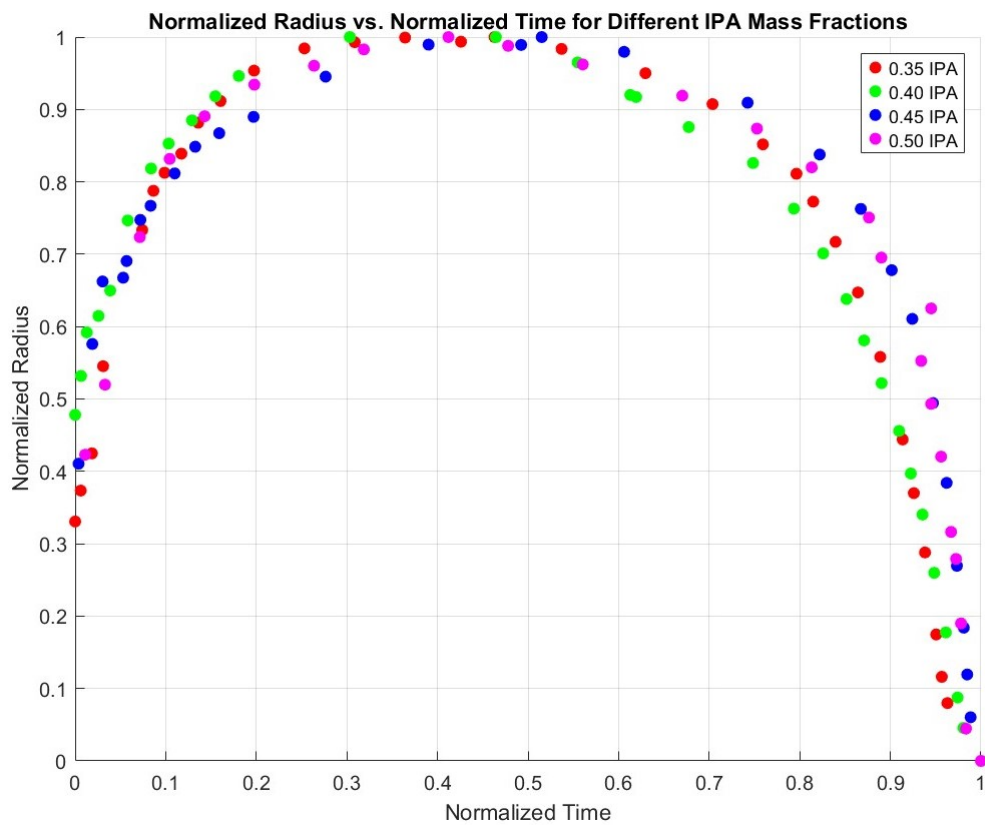


Figure 12: r_{nm} vs. t graph for various IPA mass fractions

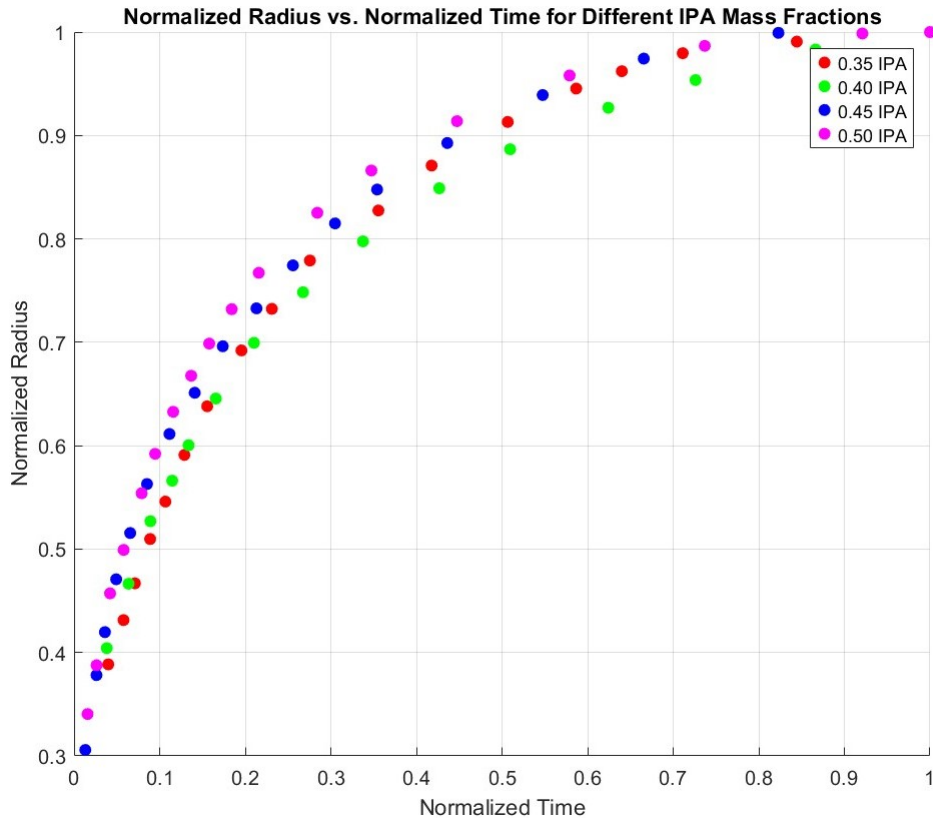


Figure 13: r_{no} vs. t graph for various IPA mass fractions

References

- [1] Lee, S., Lee, H.-Y., Lee, I.-F., & Tseng, C.-Y. (2004). Ink diffusion in water. *European Journal of Physics*, 25(2), 331–336. <https://doi.org/10.1088/0143-0807/25/2/020>
- [2] Keiser, L., Bense, H., Colinet, P., Bico, J., & Reyssat, E. (2017). Marangoni bursting: Evaporation-induced emulsification of binary mixtures on a liquid layer. *Physical Review Letters*, 118(7), 074504. <https://doi.org/10.1103/PhysRevLett.118.074504>
- [3] Socolofsky, S. A., & Jirka, G. H. (2002). *Environmental Fluid Mechanics Part I: Mass Transfer and Diffusion*. Institut für Hydromechanik, Universität Karlsruhe. Retrieved from <https://publikationen.bibliothek.kit.edu/1542004/1362>
- [4] Gradshteyn, I. S., & Ryzhik, I. M. (2007). Table of Integrals, Series, and Products (7th ed., edited by A. Jeffrey & D. Zwillinger), p. 706, Eq. 6.631-4. *Elsevier/Academic Press*. <https://archive.org/details/GradshteinI.S.RyzhikI.M.TablesOfIntegralsSeriesAndProducts>
- [5] Anonymous. (2005). APh 162 – Biological Physics Laboratory: Diffusion of solid particles confined in a viscous fluid. Retrieved from <https://api.semanticscholar.org/CorpusID:10253517>
- [6] Caretto, L. (2009). Solution of the diffusion equation. *Notes on Engineering Analysis, College of Engineering and Computer Science, Mechanical Engineering Department*. Retrieved from <http://www.csun.edu/lcaretto/me501b/diffusion.doc>
- [7] Tadmor, R., Baksi, A., Gulec, S., Jadhav, S., N'guessan, H. E., Sen, K., Somasi, V., Tadmor, M., Wasnik, P., & Yadav, S. (2019). Drops that change their mind: Spontaneous reversal from spreading to retraction. *Langmuir*, 35(48), 15734–15738. <https://doi.org/10.1021/acs.langmuir.9b02592>

- [8] Baumgartner, D. A., Shiri, S., Sinha, S., Karpitschka, S., & Cira, N. J. (2022). Marangoni spreading and contracting three-component droplets on completely wetting surfaces. *Proceedings of the National Academy of Sciences*, 119(19), e2120432119. <https://doi.org/10.1073/pnas.2120432119>
- [9] Kim, H., Muller, K., Shardt, O., Afkhami, S., & Stone, H. A. (2017). Solutal Marangoni flows of miscible liquids drive transport without surface contamination. *Nature Physics*, 13(11), 1105–1110. <https://doi.org/10.1038/nphys4214>
- [10] Jia, F., Wang, T., Peng, X., & Sun, K. (2022). Three stages of Marangoni-driven film spreading for miscible fluids. *Physics of Fluids*, 34(12), 121705. <https://doi.org/10.1063/5.0132216>
- [11] Jia, F., Peng, X., Wang, J., Wang, T., & Sun, K. (2024). Marangoni-driven spreading of a droplet on a miscible thin liquid layer. *Journal of Colloid and Interface Science*, 658, 617–626. <https://doi.org/10.1016/j.jcis.2023.12.092>
- [12] Roché, M., Li, Z., Griffiths, I. M., Le Roux, S., Cantat, I., Saint-Jalmes, A., & Stone, H. A. (2014). Marangoni flow of soluble amphiphiles. *Physical Review Letters*, 112(20), 208302. <https://doi.org/10.1103/PhysRevLett.112.208302>
- [13] Suciú, D. G., Smigelschi, O., & Ruckenstein, E. (1970). The spreading of liquids on liquids. *Journal of Colloid and Interface Science*, 88(4), 535–540.
- [14] Durey, G., Kwon, H., Magdelaine, Q., Casiulis, M., Mazet, J., Keiser, L., Bense, H., Bico, J., Colinet, P., & Reyssat, E. (2017). Marangoni bursting: Evaporation-induced emulsification of a two-component droplet. *70th Annual Meeting of the APS Division of Fluid Dynamics - Gallery of Fluid Motion*. <https://doi.org/10.1103/APS.DFD.2017.GFM.V0020>