PHY 312 **Due by:**
March 11, 2025 **Homework 3** March 24, 11 am

Problem 1

(Zettili Q.7.6) Consider the wave function of a particle

$$\Psi(r,\theta,\phi) = (\sqrt{2}x + \sqrt{2}y + z)f(r),$$

where $f(r)$ is a spherically symmetric function.

(a) Is $\Psi(r,\theta,\phi)$ an eigenfunction of L^2? If so, what is the eigenvalue?

(b) What are the probabilities for the particle to be found in the states $m_l = -1$, $m_l = 0$, and $m_l = 1$?

(c) If $\Psi(r,\theta,\phi)$ is an energy eigenfunction with eigenvalue E and if $f(r) = 3r^2$, find the expression of the potential $V(r)$ to which this particle is subjected.

Problem 2

Suppose a half-integer l-value, say $\frac{1}{2}$, were allowed for orbital angular momentum. From

$$\hat{\mathbf{L}}_+ Y_{1/2,1/2}(\theta,\phi) = 0,$$

we may deduce, as usual,

$$Y_{1/2,1/2}(\theta,\phi) \propto e^{i\phi/2}\sqrt{\sin\theta}.$$

Now try to construct $Y_{1/2,-1/2}(\theta,\phi)$ by **(a)** applying L_- to $Y_{1/2,1/2}(\theta,\phi)$; and **(b)** using $L_- Y_{1/2,-1/2}(\theta,\phi) = 0$. Show that the two procedures lead to contradictory results. (This gives an argument against half-integer l-values for orbital angular momentum.)

Problem 3

A D_2 molecule at 30 K, at $t = 0$, is known to be in the state

$$\psi(\theta,\phi,0) = \frac{3Y_1^1 + 4Y_7^3 + Y_7^1}{\sqrt{26}}.$$

(a) What values of L and L_z will measurement find and with what probabilities will these values occur?

(b) What is $\psi(\theta,\phi,t)$?

(c) What is $\langle E \rangle$ for the molecule (in eV) at $t > 0$? (*Note:* For the purely rotational states of D_2, assume that $\hbar/4\pi I_c = 30.4\ \text{cm}^{-1}$.)

Homework 3

Problem 4

A particle of spin $\frac{1}{2}$ is in a d-state of orbital angular momentum (i.e., $l = 2$).

(a) Work out the coupling of the spin and orbital angular momenta of this particle, and find all the states and the corresponding Clebsch–Gordan coefficients.

(b) If its Hamiltonian is given by

$$H = a + b\hat{L} \cdot \hat{S} + c\hat{L}^2,$$

where a, b, and c are numbers, find the values of the energy for each of the different states of total angular momentum. Express your answer in terms of a, b, and c.

Problem 5

(Shankar Q.12.5.3)

(a) Show that $\langle J_x \rangle = \langle J_y \rangle = 0$ in a state $|j, m\rangle$.

(b) Show that in these states:

$$\langle J_x^2 \rangle = \langle J_y^2 \rangle = \frac{1}{2}\hbar^2[j(j+1) - m^2].$$

(Use symmetry arguments to relate $\langle J_x^2 \rangle$ to $\langle J_y^2 \rangle$.)

(c) Check that $\Delta J_x \cdot \Delta J_y$ from part (b) satisfies the inequality imposed by the uncertainty principle.

(d) Show that the uncertainty bound is saturated in the state $|j, \pm j\rangle$.

Problem 6

Consider a system of two particles. Particle 1 has spin $s_1 = \frac{1}{2}$ and particle 2 has spin $s_2 = \frac{1}{2}$. The total spin of the system is given by

$$S = S_1 + S_2.$$

(a) List all the possible uncoupled basis states $|s_1, s_2, m_1, m_2\rangle$.

(b) Identify the stretched state $|s_1, s_2, m_1, m_2\rangle$.

(c) Starting with the stretched state, generate all the coupled basis states $|S, M\rangle$ using the lowering operator and the orthogonality condition.

(d) From the results in (c), construct the Clebsch-Gordan table for this system.