Quantum Optics: HW4 by Muhammad Sabieh Anwar

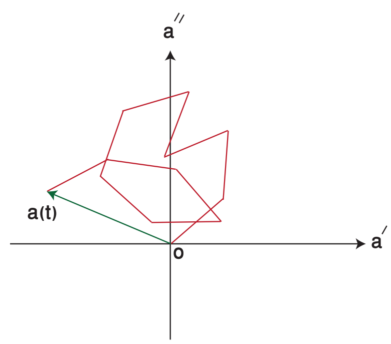
Please submit before 10 am, Monday, 28 April 2025; only by the channel communicated

1. For thermal radiation, the probability distribution for n photons in a single mode ω at temperature T is given by

$$P(n) = \frac{e^{-n\hbar\omega/k_B t}}{\sum_n e^{-n\hbar\omega/k_B t}}. \tag{1}$$

Use this to find $(\Delta n)^2$ and show that it has a "shot noise" component plus an "extra noise" component.

2. (a) Consider a random walk in two dimensions. After k steps, what is the average value of the square of the distance a, i.e. find $\langle a^2 \rangle$?



(b) Suppose $\vec{a} = a' + ia''$ where a' and a'' are real numbers. Both a' and a'' are Gaussian variables, each with a standard deviation σ. Find $P(a)$ which should be normalzied.

(c) Based on the answer to the previous two parts, find $P(a)$ in terms of k.

(d) For chaotic light with measurement time $T \gg \tau_c$ (= coherence time), in class we showed that

$$\begin{aligned} E(t) &= E_o a(t) e^{i\phi(t)}\ e^{-i\omega t} && (2)\\ &= \textstyle\sum_i E_o e^{i\phi_i(t)}\ e^{-i\omega t} && (3) \end{aligned}$$

where $a(t)$ and $\phi(t)$ are random variables. In the long time limit, under the ergodicity hypothesis,

$$\begin{aligned} \bar{I} &= \textstyle\sum_{i=1}^{k} \left| E_o a_i(t) e^{i\phi(t)} \right|^2 && (4)\\ &= \tfrac{1}{2} c\varepsilon_0 E_o^2 k && (5) \end{aligned}$$

From Equation (2), we also obtain at time t the instantaneous intensity,

$$\bar{I}(t) = \frac{1}{2} c\varepsilon_0 E_0^2 a^2(t). \quad (6)$$

From the answer in (c), find $P(\bar{I}(t))$.

(e) Finally, find $P(N)$ for an arbitrary detection efficiency ξ. Plot for some representative values of (say $\xi T = 0.6$, $\bar{I} = 1$). [This would require finding an integral.]

(f) Show that your result for $P(N)$ matches that of a thermal state for the detection efficiency ξ.

3. The previous question dealt with chaotic light and $T \gg \tau_c$. In this question, we build photon statistics for arbitrary randomly fluctuating light for any measurement times.

For coherent light, we already know that:

$$P(N) = \frac{\bar{N}^N\ e^{-\bar{N}}}{N!} \quad (7)$$

from which we extrapolate,

$$P(N;t,T) = \frac{\left(\xi \bar{I}(t,T)T\right)^N e^{\xi \bar{I}(t,T)T}}{N!}. \tag{8}$$

For a stationary state, invoking the ergodic hypothesis we can write this equation as,

$$P(N;t,T) = \left\langle e^{-\xi \bar{I}(t,T)T} \right\rangle \frac{\left(e^{\xi \bar{I}(t,T)T}\right)^N}{N!}. \tag{9}$$

(a) From $P(N;t,T)$ find $\bar{N}$.

(b) From $P(N;t,T)$ find $\langle N^2 \rangle$.

(c) From these results, find (ΔN^2).

(d) From the result in (c), show that $(\Delta N^2) = \bar{N}$ *only for coherent light*.