Quantum Optics: HW4 by Muhammad Sabieh Anwar

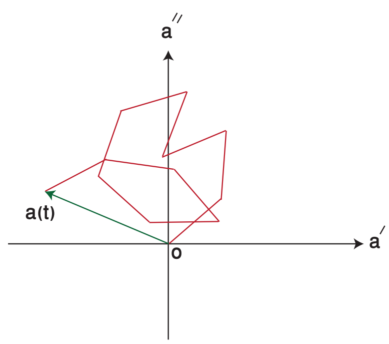
Please submit before 10 am, Monday, 28 April 2025; only by the channel communicated

1. For thermal radiation, the probability distribution for n photons in a single mode ω at temperature T is given by

$$P(n) = \frac{e^{-n\hbar\omega/k_B t}}{\sum_n e^{-n\hbar\omega/k_B t}}. \tag{1}$$

Use this to find $(\Delta n)^2$ and show that it has a "shot noise" component plus an "extra noise" component.

2. (a) Consider a random walk in two dimensions. After k steps, what is the average value of the square of the distance a, i.e. find $\langle a^2 \rangle$?



(b) Suppose $\vec{a} = a' + ia''$ where a' and a'' are real numbers. Both a' and a'' are Gaussian variables, each with a standard deviation σ. Find $P(a)$ which should be normalzied.

(c) Based on the answer to the previous two parts, find $P(a)$ in terms of k.

(d) For chaotic light with measurement time $T \gg \tau_c$ (= coherence time), in class we showed that

$$E(t) = E_o a(t) e^{i\phi(t)} \ e^{-i\omega t} \tag{2}$$

$$= \sum_i E_o e^{i\phi_i(t)} \ e^{-i\omega t} \tag{3}$$

where $a(t)$ and $\phi(t)$ are random variables. In the long time limit, under the ergodicity hypothesis,

$$\bar{I} = \sum_{i=1}^{k} \left| E_o a_i(t) e^{i\phi(t)} \right|^2 \tag{4}$$

$$= \tfrac{1}{2} c\varepsilon_0 E_o^2 k \tag{5}$$

From Equation (2), we also obtain at time t the instantaneous intensity,

$$\bar{I}(t) = \frac{1}{2} c\varepsilon_0 E_0^2 a^2(t). \tag{6}$$

From the answer in (c), find $P(\bar{I}(t))$.

(e) Finally, find $P(N)$ for an arbitrary detection efficiency ξ. Plot for some representative values of (say $\xi T = 0.6$, $\bar{I} = 1$). [This would require finding an integral.]

(f) Show that your result for $P(N)$ matches that of a thermal state for the detection efficiency ξ.

3. The previous question dealt with chaotic light and $T \gg \tau_c$. In this question, we build photon statistics for arbitrary randomly fluctuating light for any measurement times.

For coherent light, we already know that:

$$P(N) = \frac{\bar{N}^N \ e^{-\bar{N}}}{N!} \tag{7}$$

from which we extrapolate for chaotic light,

$$P(N;t,T) = P(N;T) = \frac{\left(\xi \bar{I}(t,T)T\right)^N e^{-\xi \bar{I}(t,T)T}}{N!}. \tag{8}$$

For a stationary form of light, we can invoke the ergodic hypothesis and take the average over all times t, which can be converted into an average over intensities. Hence the probability becomes independent of time t allowing us to write,

$$P(N;T) = \left\langle e^{-\xi \bar{I}(t,T)T} \frac{\left(\xi \bar{I}(t,T)T\right)^N}{N!} \right\rangle. \tag{9}$$

(a) From $P(N;T)$ find $\bar{N}$.

(b) From $P(N;T)$ find $\langle N^2 \rangle$.

(c) From these results, find (ΔN^2).

(d) From the result in (c), show that $(\Delta N^2) = \bar{N}$ *only for coherent light*.