Historical Altitude Measurements

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Abstract

Historical astronomical instruments, such as the quadrant and the triquetrum, were integral to the development of observational astronomy from the time of Ptolemy through the Islamic Golden Age and the European Renaissance. This study investigates the practical accuracy and educational utility of these tools by designing and constructing functional replicas using CNC engraving techniques. To evaluate their performance, solar altitude measurements were recorded over a five-hour period and compared with precise data from the Stellarium software. While the quadrant demonstrated high alignment with Stellarium's computational models, the triquetrum exhibited a zero offset of approximately +6° due to structural flexing of its wooden stand. Potential design refinements using rigid materials are discussed, and it is concluded that these instruments can serve as effective pedagogical tools for introducing students to the history of science and the fundamentals of celestial observation, especially in medieval times.

Introduction

The altitude of a celestial object is defined as its height above the horizon (the apparent line where the Earth and sky meet). The purpose of this study was to investigate ancient astronomical instruments used to measure altitude; specifically the quadrant and the triquetrum (also called the parallactic ruler). These ancient tools can be traced back to the time of the 2nd-century astronomer, Claudius Ptolemy, who famously authored the *Almagest*. These tools were later also used by other Western astronomers in the 16th-century such as Nicolaus Copernicus and Tycho Brahe [1, 2].

Such tools were also widely present in the Islamic world, especially within observatories such as the Maragha Observatory (located in present-day Iran) [3] and the

Istanbul Observatory a few centuries later [4]. The time period ranging from the 8th to the 15th centuries in the Islamic world was extremely fruitful for astronomy. Motivated by an obligation to determine the times of the five daily prayers and the direction of the *qibla* (direction towards the Kabbah in Makkah), Muslim scholars led developments in instrumentation, such as variants of the quadrant (also called *rub'*) and the triquetrum (called al-iḍāda al-ṭawīla - the long alidade) and refined theories that would benefit later scientists, including Copernicus [5, 6].

With such a rich history behind these instruments, this study aimed to determine how well the quadrant and triquetrum performed compared to modern-day methods of altitude determination.

Setups

Figure 1: The quadrant (Q) and triquetrum (T).

The quadrant and triquetrum used for this experiment were designed and constructed from scratch. The design of the quadrant was a simplified version based on a mariner's quadrant available in Royal Museums Greenwich [7], while the design of the triquetrum was based on a Polish build from 1948 [8].

Blueprints for both instruments were outlined using `Google Drawings`, and are available for public view [9, 10]. With the exception of minor differences, the final builds closely follow the dimensions specified in the blueprints.

The wood primarily used in both builds was medium-density fiberboard (MDF). For the quadrant, the scale was created using Solid Edge [11] and was engraved on layer of paper on top of the wooden structure using a CNC machine. Additionally, a small hole was drilled at the top corner for the attachment of a plumb-line.

For the triquetrum, metal hinges were attached between the supporting arm and the other arms, and the slider on the moving arm employed a locking mechanism (which also doubled as a sight). It was also important to have the triquetrum stand aligned with the zenith (the imaginary point in the sky directly overhead an observer). To help keep the stand straight and reduce bending in the wood, a string was tied to the top of the stand and the bottom of the base, and a plank of wood was attached at an angle between the stand and the base. Finally, for the scale of the ruler arm, a measure tape was attached directly on the arm.

Methodology

The following subsections discuss the steps required to operate the quadrant and the triquetrum.

Quadrant

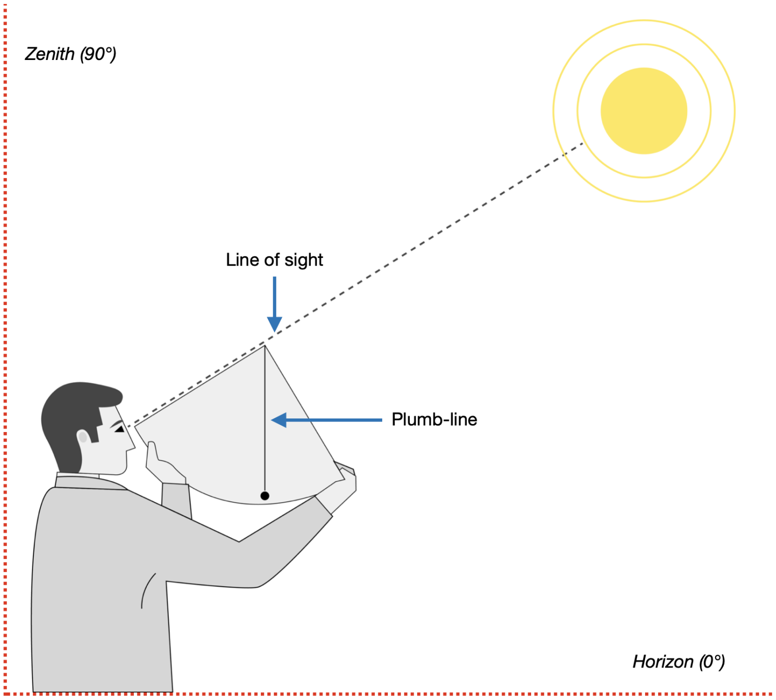

Figure 2: Labeled illustration of the quadrant.

1. Aim the quadrant directly towards the celestial object of interest from the edge where 0° is marked. Ensure that your aim is stable.

2. The plumb-line will align with a marking on the quadrant's scale. This reading can be recorded with the help of a phone camera, or can be manually noted. A friend's help may be useful at this stage.
3. The reading obtained is known as the zenith distance (a misnomer for the angular separation between the zenith and your celestial object). Subtract the reading from 90° to obtain the altitude of your object of interest.

Triquetrum

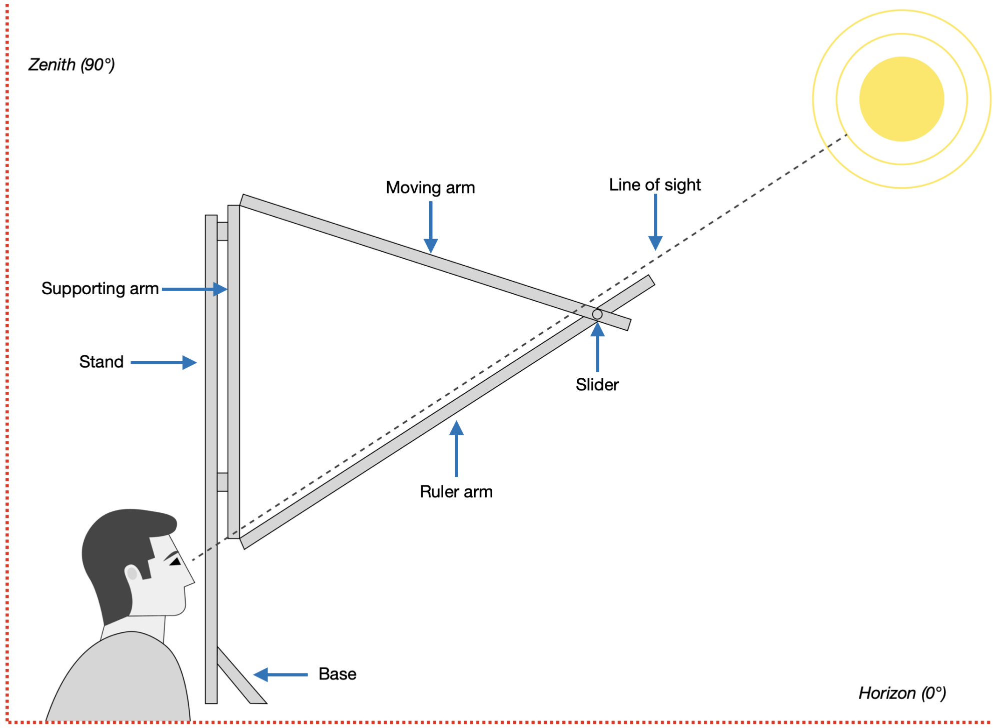

Figure 3: Labeled illustration of the triquetrum.

1. Position your line of sight along the ruler arm (from the bottom hinge of the triquetrum).
2. Using the lock as a sight, adjust the ruler arm and moving arm until the object aligns completely with the slider. Lock the slider into place.
3. Note the reading from the point between the slider and the ruler arm. Call this reading R.
4. Use the cosine rule to determine the zenith distance Z. In this setup, the triquetrum forms an isosceles triangle, so the equation simplifies to:

$$Z = \arccos(\frac{R}{2L})$$

Here, L refers to the constant lengths of the supporting arm and the moving arm (both of which are 90 cm for this setup).

5. Just as you have done with the quadrant, subtract the zenith distance from 90° to obtain the altitude of your object of interest.

Observation and results

To test both the quadrant and the triquetrum, the Sun was chosen as a target for a nearly five-hour observation (from 11:20 AM to 4:00 PM on 11th December 2025) on the campus of Lahore University of Management Sciences (located near DHA Phase 5, Lahore). Readings from both the quadrant and triquetrum were taken every 20 minutes, leading to a total of 15 readings.

It is important to mention that **the Sun must never be observed directly without a certified solar filter. Observing the Sun without the necessary protection puts your eye at risk of severe damage.**

Uncertainties

Type B uncertainties were taken into account for our readings. For the two analog instruments, uncertainty is taken as $\Delta/\sqrt{6}$, where Δ is half of the least count of the instrument.

As each measurement with the quadrant and triquetrum was not instantaneous, uncertainty in time was taken as 2.5 minutes. Meanwhile, the least count of the quadrant's scale was 1°, so standard uncertainty was taken as 0.2°.

For the triquetrum, the least count of the ruler's scale was 0.1 cm, standard uncertainty is taken as 0.02 cm. Uncertainty in the known lengths of the supporting and moving arms was also taken as 0.02 cm. This leads to a case of error propagation (converting error in centimeters to error in degrees). This can be calculated via the formula:

$$\sigma_Z \approx \sqrt{(\tfrac{dZ}{dR}\sigma_R)^2 + (\tfrac{dZ}{dL}\sigma_L)^2}$$

As the zenith distance Z is simply the complement of the altitude, its uncertainty σ_Z would be equal to the uncertainty in the triquetrum's altitude.

To represent a 95% confidence interval (expanded uncertainty) in our graph, we doubled the calculated values for the final quadrant (θ_Q) and triquetrum (θ_T) altitude uncertainties.

Results

With the obtained data and calculated uncertainties, plots for both the quadrant and triquetrum were generated. The plots were compared with data from the popular astronomy software Stellarium [12].

Time (24-hour)*	θ_Q (°)	$\theta_T - 6$ (°)	Stellarium reference (°)
11 : 20	34 ± 0.4	34.79 ± 0.03	34.88
11 : 40	37 ± 0.4	35.85 ± 0.03	35.41
12 : 00	37 ± 0.4	35.42 ± 0.03	35.52
12 : 20	36 ± 0.4	35.38 ± 0.03	35.21
12 : 40	36 ± 0.4	33.46 ± 0.03	34.49
13 : 00	35 ± 0.4	33.71 ± 0.03	33.38
13 : 20	32 ± 0.4	30.35 ± 0.02	31.89
13 : 40	31 ± 0.4	29.45 ± 0.02	30.06
14 : 00	28 ± 0.4	27.71 ± 0.02	27.91
14 : 20	26 ± 0.4	25.86 ± 0.02	25.48
14 : 40	24 ± 0.4	23.23 ± 0.02	22.80
15 : 00	20 ± 0.4	21.42 ± 0.02	19.80
15 : 20	17 ± 0.4	18.38 ± 0.02	16.71
15 : 40	13 ± 0.4	16.82 ± 0.02	13.56
16 : 00	10 ± 0.4	13.27 ± 0.02	10.08

Table 1: Altitude values of the Sun obtained from quadrant (θ_Q), triquetrum (θ_T; adjusted by -6 degrees) and Stellarium on December 11th, 2025. *Time has an uncertainty of 2.5 minutes.

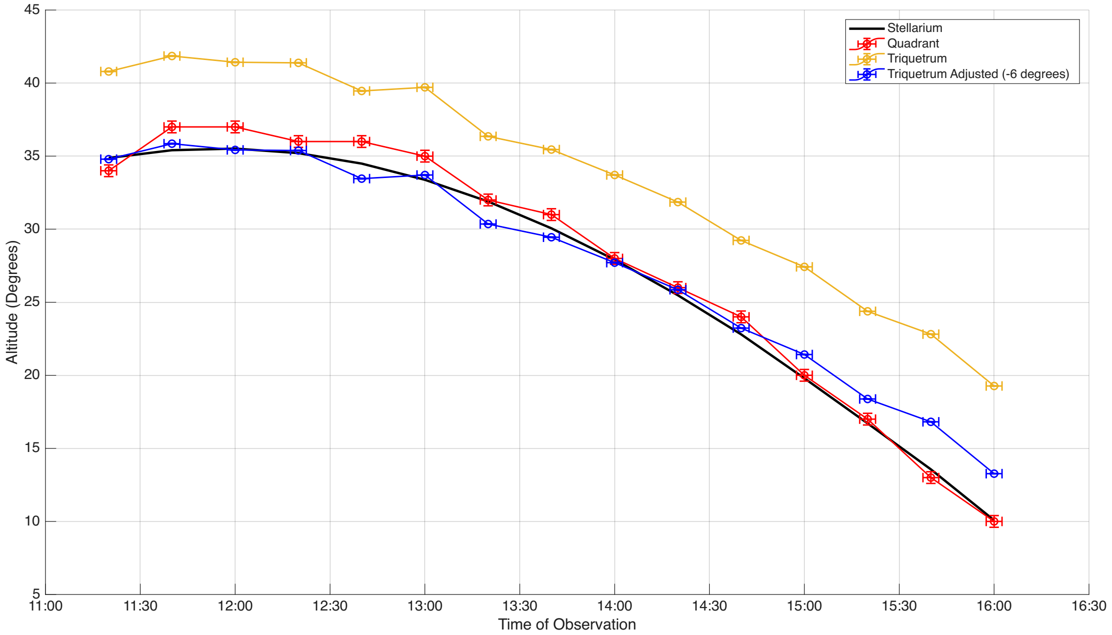

Figure 4: Graph of altitude values of the Sun versus time (December 11th, 2025). For triquetrum altitude values, the uncertainty is smaller than the marker size.

The data obtained from the quadrant matches closely to the data from Stellarium. However, this initially does not seem to be the case for the triquetrum, due to a significant gap between its plot and the plot of Stellarium. This gap can be attributed to a zero offset in the triquetrum. Due to the weight of the rest of the instrument, the triquetrum's stand leans forward, and therefore does not exactly align with the zenith. For the sake of comparison, another plot for the triquetrum is shown in Figure 4, where the zero offset is assumed to be -6°.

Discussion and conclusion

Despite the simplistic setups of the quadrant and triquetrum, they are able to provide data comparable to those provided through modern techniques. However, improvements can be made. In the case of the triquetrum, metal would be a preferable choice of material over wood, as it provide the instrument a sturdy structure and greatly reduce the zero offset currently present in the wood-based triquetrum. For the quadrant, white arcylic can be used instead of wood, improving aesthetics and providing a more rigid structure for similar weight. For both instruments, dedicated sights would need to be attached to ensure easier viewing. Implementing these improvements would enable these versions of the instruments to be more easily usable on other celestial objects, such as the Moon and the planets.

The development of such instruments would be extremely useful in the education sector, due to their historical and scientific significance. Through the use of such instruments in public science fairs and exhibitions, the public can be more educated about how astronomical observations were carried out in ancient times. In schools, these instruments can serve as a introduction to astronomy for many students, providing them history on the field and giving them an opportunity to learn basic observational skills before they progress to the more sophisticated telescope setups of today.

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