

Dispersion Relation for Phonons in 2 D

Along Γ - X line

$$\text{In[1]:= } \alpha = 1; \beta = \alpha / 2;$$

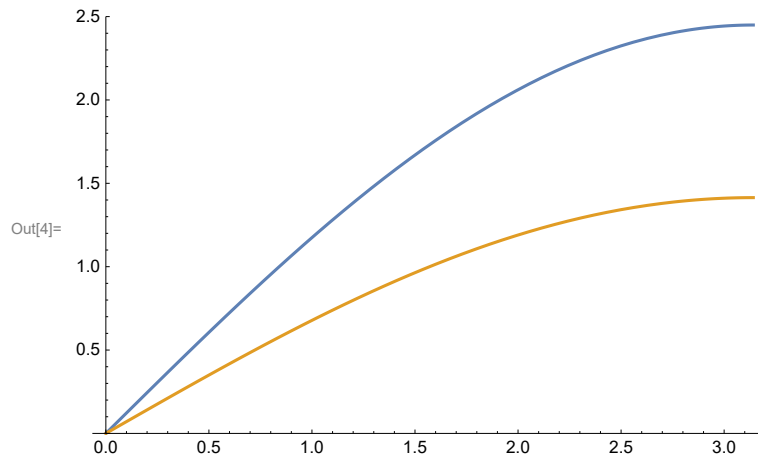
$$\text{In[2]:= } \omega_1 = \text{Sqrt}[2 * (1 - \text{Cos}[kx]) * (\alpha + \beta)]$$

$$\text{Out[2]:= } \sqrt{3} \sqrt{1 - \text{Cos}[kx]}$$

$$\text{In[3]:= } \omega_2 = \text{Sqrt}[2 * \beta * (1 - \text{Cos}[kx])]$$

$$\text{Out[3]:= } \sqrt{1 - \text{Cos}[kx]}$$

$$\text{In[4]:= } \text{q2} = \text{Plot}[\{\omega_1, \omega_2\}, \{kx, 0, \text{Pi}\}, \text{PlotRange} \rightarrow \{0, 2.5\}]$$



Along X-M line

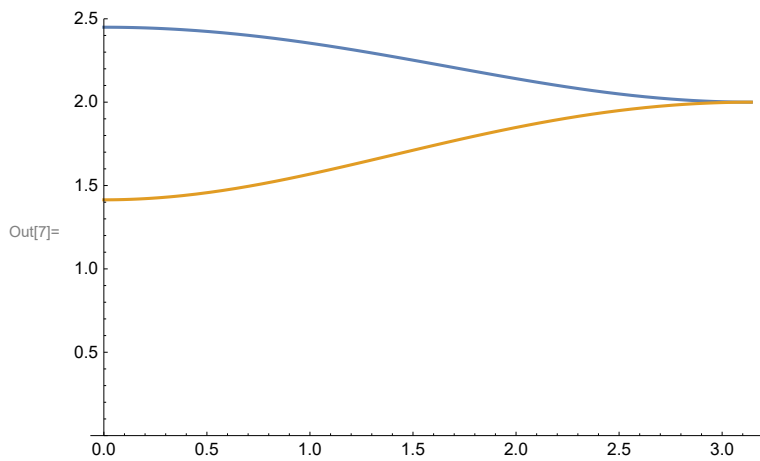
$$\text{In[5]:= } \omega_4 = \text{Sqrt}[4 * \alpha + 2 * \beta * (1 + \text{Cos}[ky])]$$

$$\text{Out[5]:= } \sqrt{5 + \text{Cos}[ky]}$$

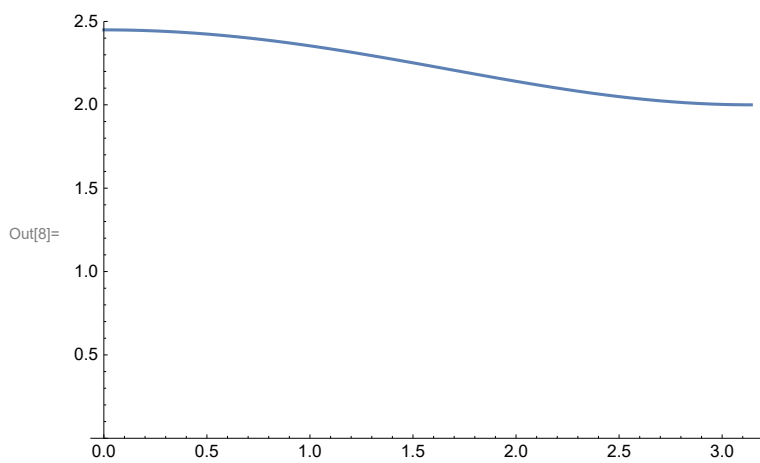
$$\text{In[6]:= } \omega_3 = \text{Sqrt}[2 * (\alpha * (1 - \text{Cos}[ky]) + \beta * (1 + \text{Cos}[ky]))]$$

$$\text{Out[6]:= } \sqrt{2} \sqrt{1 - \text{Cos}[ky] + \frac{1}{2} (1 + \text{Cos}[ky])}$$

In[7]:= **q3 = Plot[{ ω_4 , ω_3 }, {ky, 0, Pi}, PlotRange -> {0, 2.5}]**



In[8]:= **Plot[{ ω_4 }, {ky, 0, Pi}, PlotRange -> {0, 2.5}]**



Along Γ -M line

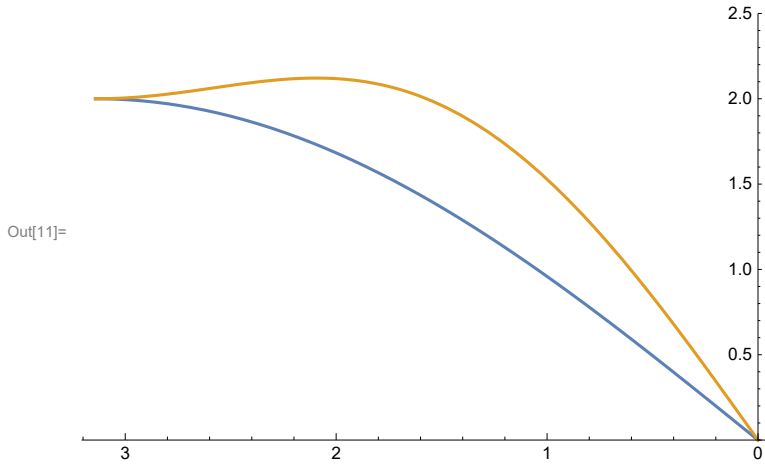
In[9]:= **$\omega_5 = \text{Sqrt}[2 * (\alpha * (1 - \text{Cos}[kx]))]$**

Out[9]= $\sqrt{2} \sqrt{1 - \text{Cos}[kx]}$

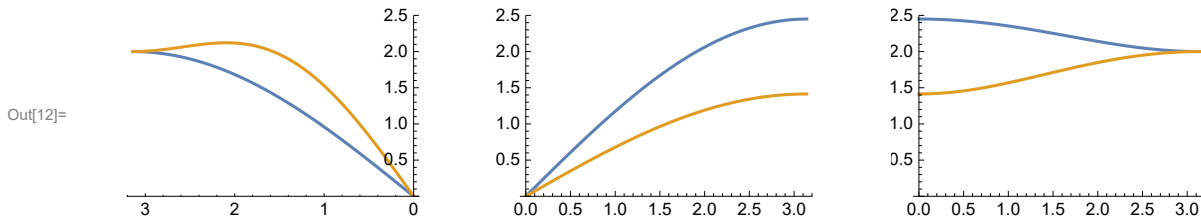
In[10]:= **$\omega_6 = \text{Sqrt}[2 * (\alpha * (1 - \text{Cos}[kx]) + \beta * (1 - \text{Cos}[2 * kx]))]$**

Out[10]= $\sqrt{2} \sqrt{1 - \text{Cos}[kx] + \frac{1}{2} (1 - \text{Cos}[2 kx])}$

```
In[11]:= q1 = Plot[{ω5, ω6}, {kx, 0, Pi},
  ScalingFunctions -> {"Reverse", Identity}, PlotRange -> {0, 2.5}]
```



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In[12]:= GraphicsRow[{q1, q2, q3}]
```



```
In[13]:= a = -ω^2 + 2 * (α * (1 - Cos[kx]) + β * (1 - Cos[kx] * Cos[ky]))
```

```
Out[13]= -ω^2 + 2 (1 - Cos[kx] + 1/2 (1 - Cos[kx] Cos[ky]))
```

```
In[14]:= b = 2 * β * Sin[kx] * Sin[ky]
```

```
Out[14]= Sin[kx] Sin[ky]
```

```
In[15]:= cc = -ω^2 + 2 * (α * (1 - Cos[ky]) + β * (1 - Cos[kx] * Cos[ky]))
```

```
Out[15]= -ω^2 + 2 (1 - Cos[ky] + 1/2 (1 - Cos[kx] Cos[ky]))
```

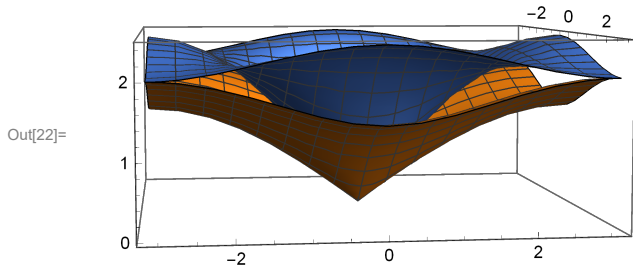
```
In[16]:= Q = {{a, b}, {b, cc}}
```

```
Out[16]= {{-ω^2 + 2 (1 - Cos[kx] + 1/2 (1 - Cos[kx] Cos[ky])), Sin[kx] Sin[ky]},
  {Sin[kx] Sin[ky], -ω^2 + 2 (1 - Cos[ky] + 1/2 (1 - Cos[kx] Cos[ky]))}}
```

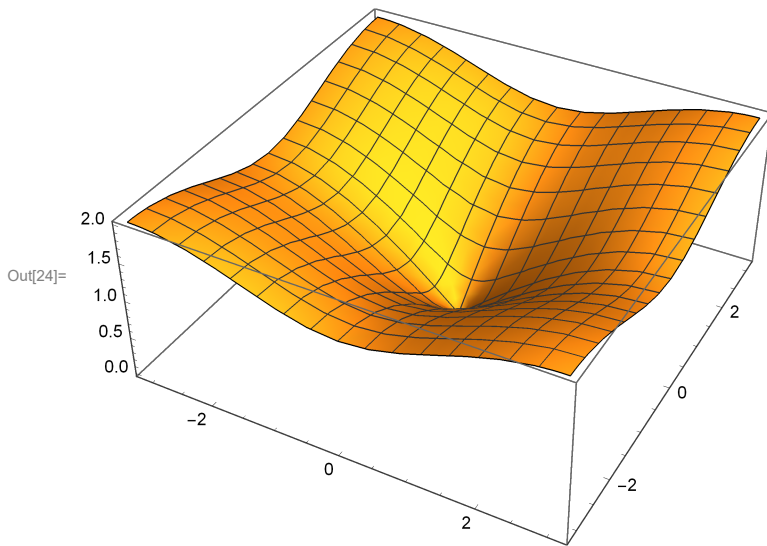
```
In[19]:= Solve[FullSimplify[Det[Q]] == 0, ω]
```

```
Out[19]= {{ω -> -√(2 - Cos[kx] - Cos[ky])}, {ω -> √(2 - Cos[kx] - Cos[ky])},
  {ω -> -√(4 - Cos[kx] - Cos[kx - ky] - Cos[ky] - Cos[kx + ky])},
  {ω -> √(4 - Cos[kx] - Cos[kx - ky] - Cos[ky] - Cos[kx + ky])}}
```

In[22]:= `Plot3D[{ $\sqrt{2 - \text{Cos}[kx] - \text{Cos}[ky]}$, $\sqrt{4 - \text{Cos}[kx] - \text{Cos}[kx - ky] - \text{Cos}[ky] - \text{Cos}[kx + ky]}$ }, {kx, -Pi, Pi}, {ky, -Pi, Pi}]`



In[24]:= `Plot3D[{ $\sqrt{2 - \text{Cos}[kx] - \text{Cos}[ky]}$ }, {kx, -Pi, Pi}, {ky, -Pi, Pi}]`



In[23]:= `Plot3D[$\sqrt{4 - \text{Cos}[kx] - \text{Cos}[kx - ky] - \text{Cos}[ky] - \text{Cos}[kx + ky]}$, {kx, -Pi, Pi}, {ky, -Pi, Pi}]`

