Assignment 6

Free Electrons, Heat Capacity
Please no AI-generated solutions. Would be your loss!

Question 1

Derive the density of states $D(E)$ for free electrons:

(a) confined to a $1D$ chain.

(b) confined to a $2D$ plane.

Question 2

For spin-1/2 electrons, derive a expression on the magnetization M and consequently $\chi = \frac{M\mu_0}{B}$ (the susceptibility) and indicate that it is largely independent of temperature.

Question 3

Calculate the heat capacity C_V for a gas of free electrons in $2D$. How does C_V depend on temperature T in the low temperature regime?

Question 4

Estimate the Fermi temperature T_F of:

(a) liquid ^{3}He ($\rho = 81$ kgm^{-3}).

(b) neutrons in a neutron state ($\rho = 10^{17}$ kg m^{-3}).

Question 5

(a) Find the total energy U for free electrons at 0 K in a metal of volume V?

(b) Given $P = -\frac{\partial U}{\partial V}\Big|_N$, find the pressure of this electron gas.

(c) We have 2 kg of ^{239}Pu (plutonium) metal ($\rho = 1.9 \times 10^4$ kgm^{-3}). This is a spherical shell intended to be used as an explosive device. A conventional explosive is required to implode this shell, reducing its volume. What change in energy U is achieved if the 2 kg spherical shell is shrunk to half its volume? The energy yield of a conventional explosive such as dynamite is 4 MJ kg-1. How much dynamite is needed to achieve the calculated ΔU? Assume 1 Pu atom gives 1 electron.

(d) For a brown dwarf of mass M, radius R, show that the total gravitational potential energy is:

$$U_{\text{grav}} = -\frac{3}{5}\frac{GM^2}{R}.$$

The total energy is:

$$U = U_e - U_{\text{grav}},$$

where U_e is the energy associated with fee electrons. For the equilibrium condition $\frac{\partial U}{\partial V} = 0$, find the relationship between mass M and radius R of the brown dwarf.