Exploration of Electronic Chaos: A Study of Nonlinear Dynamics and Chua's Circuit

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Abstract

Chua's circuit was introduced by Leon O. Chua in 1983 as a simple physical system capable of generating chaotic oscillations. This study explores the generation of deterministic chaos by integrating the mathematical modeling of nonlinear dynamics with the physical implementation of the circuit. The governing ordinary differential equations were solved numerically in Python, while experimental validation was achieved using a constructed circuit featuring a piecewise-linear nonlinear resistor. Observations of the double-scroll attractor illustrated the transition from simple periodic oscillations to complex chaotic behavior, characterized by extreme sensitivity to initial conditions. This behavior comes from the internal workings of the circuit rather than random noise, which confirms that the system is a practical way to study how chaos works.

1 Introduction

Classical physics and engineering typically describe systems through linear models, where output remains directly proportional to input, which facilitates predictable behavior. Nonlinearity occurs when the system response does not follow a simple linear relationship with the input, which causes the system to behave in ways that standard methods cannot predict.

One such behavior is chaos, which occurs when a system follows strict mathematical rules yet remains unpredictable due to its extreme sensitivity to initial conditions. Studying nonlinear systems helps explain complex processes, ranging from population dynamics to secure communication protocols. Chua's circuit serves as the standard electronic model for studying chaotic behavior. By replacing traditional linear components with a nonlinear element, the circuit generates complex, self-similar dynamics known as the double-scroll attractor.

This report presents an experimental exploration of electronic chaos. Through mathematical modeling and the practical implementation of Chua's circuit, the work demonstrates how adjustments to system parameters trigger a transition from steady-state stability to chaotic oscillation. Finally, an analysis of phase portraits and time-domain data validates the deterministic nature of chaos and highlights the importance of nonlinearity in modern electronic systems.

2 Mathematical modeling

Chua's circuit, shown in Fig. 1, is a nonlinear electronic circuit that serves as a basic system capable of producing chaotic oscillations. It consists of two capacitors, one inductor, one linear resistor, and a single nonlinear element, Chua's diode [1, 2].

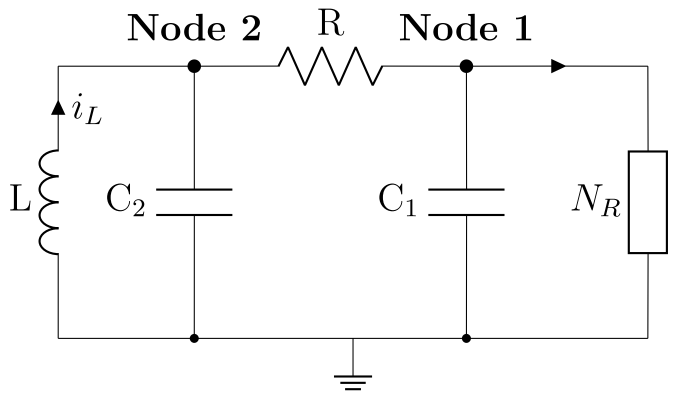


Figure 1: Schematic diagram of Chua's Circuit.

The dynamic evolution of Chua's circuit follows a system of three coupled ordinary differential equations. The analysis first isolates the node corresponding to capacitor C_1, which interfaces with coupling resistor R and the nonlinear active resistor N_R. Applying Kirchhoff's current law at the V_{C_1} node yields:

$$i_R = i_{C_1} + i_{NR}, \tag{1}$$

$$i_{C_1} = i_R - i_{NR}. \tag{2}$$

Substituting the standard constitutive branch relations where $i_{C_1} = C_1 \frac{dV_{C_1}}{dt}$ and the current passing through the resistor from node 2 to node 1 is $i_R = \frac{V_{C_2} - V_{C_1}}{R}$:

$$C_1 \frac{dV_{C_1}}{dt} = \frac{V_{C_2} - V_{C_1}}{R} - i_{NR}. \tag{3}$$

Dividing the equation (3) by the capacitance C_1 isolates the first-order derivative:

$$\frac{dV_{C_1}}{dt} = \frac{1}{C_1}\left(\frac{V_{C_2} - V_{C_1}}{R}\right) - \frac{i_{NR}}{C_1}. \tag{4}$$

Factoring the resistance R across both terms on the right-hand side matches the standard dimensionless scaling layout:

$$\frac{dV_{C_1}}{dt} = \frac{1}{RC_1}(V_{C_2} - V_{C_1}) - \frac{i_{NR}R}{RC_1}. \tag{5}$$

Defining the scaling parameter $\alpha = \frac{1}{RC_1}$ and the voltage-dependent characteristic function $f(V_{C_1}) = i_{NR}R$ establishes the first state equation:

$$\frac{dV_{C_1}}{dt} = \alpha(V_{C_2} - V_{C_1} - f(V_{C_1})). \tag{6}$$

$f(V_{C1})$ in equation (6) describes the piecewise-linear relationship between the voltage across capacitor C_1 and the current through the diode [2]. Mathematically, it is defined as:

$$f(V_{C1}) = m_1 V_{C1} + \frac{1}{2}(m_0 - m_1)(|V_{C1} + 1| - |V_{C1} - 1|).$$

Figure (2) illustrates the isolated node corresponding to capacitor C_2, which links the central coupling resistor R and the parallel inductor branch L.

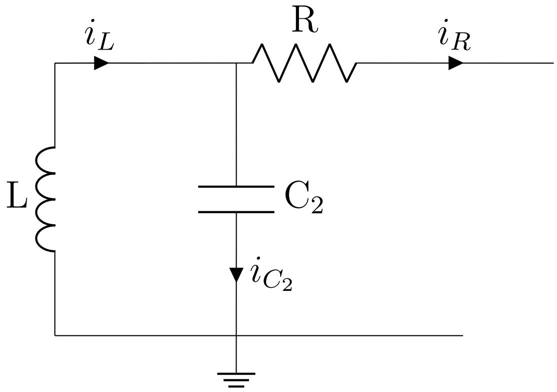


Figure 2: Isolated sub-circuit schematic focusing on the node at Capacitor C_2, detailing currents i_L, i_R, and i_{C_2}.

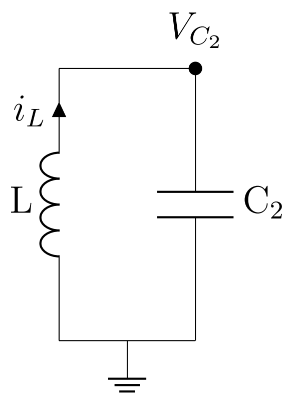
Evaluating the balance of currents entering and leaving this node via Kirchhoff's Current Law yields:

$$i_L = i_R + i_{C_2}. \tag{7}$$

Substituting the resistor and capacitor relations, and normalizing such that $R = 1$ and $C_2 = 1$, is standard in nonlinear circuit analysis; this approach emphasizes the qualitative behavior of the system without loss of generality. The second dynamic equation becomes:

$$\frac{dV_{C_2}}{dt} = V_{C_1} - V_{C_2} + i_L. \tag{8}$$

Finally, the analysis evaluates the leftmost parallel loop containing the inductive element L in parallel with the capacitor C_2, which leads to the common reference ground.



Applying Kirchhoff's Voltage Law (KVL) around the closed inductive-capacitive loop yields the balance equation:

$$V_L + V_C = 0. \tag{9}$$

Using the constitutive voltage relation of the inductor $V_L = L\frac{di_L}{dt}$ and matching the orientation of the node voltage parameter ($V_C = V_{C_2}$):

$$L\frac{di_L}{dt} = -V_{C_2}. \tag{10}$$

Isolating the time-derivative variable by dividing by the self-inductance L produces:

$$\frac{di_L}{dt} = -\frac{1}{L}V_{C_2}. \tag{11}$$

Choosing the final system dimensionless parameter notation $\beta = \frac{1}{L}$ establishes the third state equation:

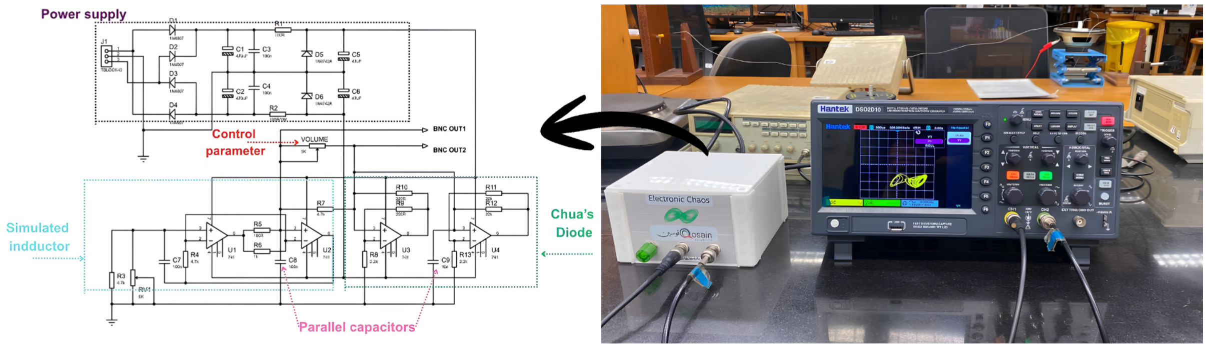
$$\frac{di_L}{dt} = -\beta V_{C_2}. \tag{12}$$

In expression (6), the parameter α scales the relationship between two capacitor nodes and the function $f(V_{C1})$ represents the piecewise-linear characteristic of the Chua diode. In equation (12), β controls the inductor's time scaling. This nonlinear element provides negative resistance, which compensates energy losses and keeps the circuit oscillating. By adjusting the resistance, the system experiences a series of bifurcations. As the control parameters cross specific values, $\alpha = 15.6$ and β =28, the system shifts from stable fixed points to limit cycles, eventually manifesting as the double-scroll attractor in three-dimensional phase space. This attractor represents the signature of deterministic chaos.

Although the trajectory of the voltages and currents appears complex and non-repeating, it remains confined within a bounded region of phase space, entirely defined by the governing differential equations(6), (8), and (12). This behavior highlights that chaos is not a result of random fluctuations but rather an inherent property of nonlinear feedback.

3 Experimental setup

Chua's circuit was studied through a combined numerical and experimental approach. Numerical simulations were carried out in Python (Google Colab), solving the circuit's coupled nonlinear differential equations using the dimensionless parameters $\alpha = 15.6$ and $\beta = 28$, with initial conditions $V_{C1} = 0.1$ V, $V_{C2} = 0$ V, and $i_L = 0$ A [1]. These values were chosen to drive the system into the "Double Scroll" regime, where chaotic dynamics are most clearly expressed. The resulting time series and phase portraits were then used to characterize the system's qualitative behavior across parameter changes.



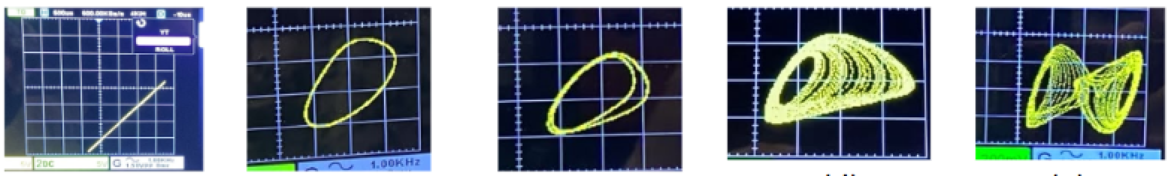
(a) (b)

Figure 3: (a) The detailed schematic diagram illustrating the three primary functional blocks of the circuit: the power supply, the simulated inductor stage and the nonlinear Chua's diode. The locations of the parallel capacitor tank (C_8, C_9) and the variable control parameter are highlighted. (b) Chua's Circuit and Digital oscilloscope in Physlab

To validate these results experimentally, the circuit was already physically constructed on PCB using 741 op-amps to realize the Chua diode, alongside two parallel capacitors, a variable resistor R serving as the control parameter, and an inductor of L, implemented via an op-amp gyrator. Power was supplied to the active components, and a digital oscilloscope, operated in X–Y mode, was used to plot phase portrait directly from the capacitor voltages. By gradually adjusting R, the circuit's dynamical response was tracked in real time, allowing the fixed-point-to-chaos transition predicted numerically to be observed and confirmed on physical hardware.

4 Results

Adjusting the variable resistance (control parameter) showed a gradual transition from simple periodic loops to a complex double-scroll pattern. The observed transition sequence was:



(a) (b) (c) (d) (e)

Figure 4: Oscilloscope observations showing the transition from periodic behavior to the double-scroll attractor as the control parameter is varied: (a) stable fixed point, (b) limit cycle, (c) period-doubling, (d) onset of chaos, and (e) double-scroll attractor.

A critical experimental observation confirmed the theoretical prediction of extreme sensitivity to initial conditions: returning the control knob to the same position did not yield the exact same pattern. This lack of reproducibility demonstrates that the system is genuinely chaotic and not merely periodic with high complexity.

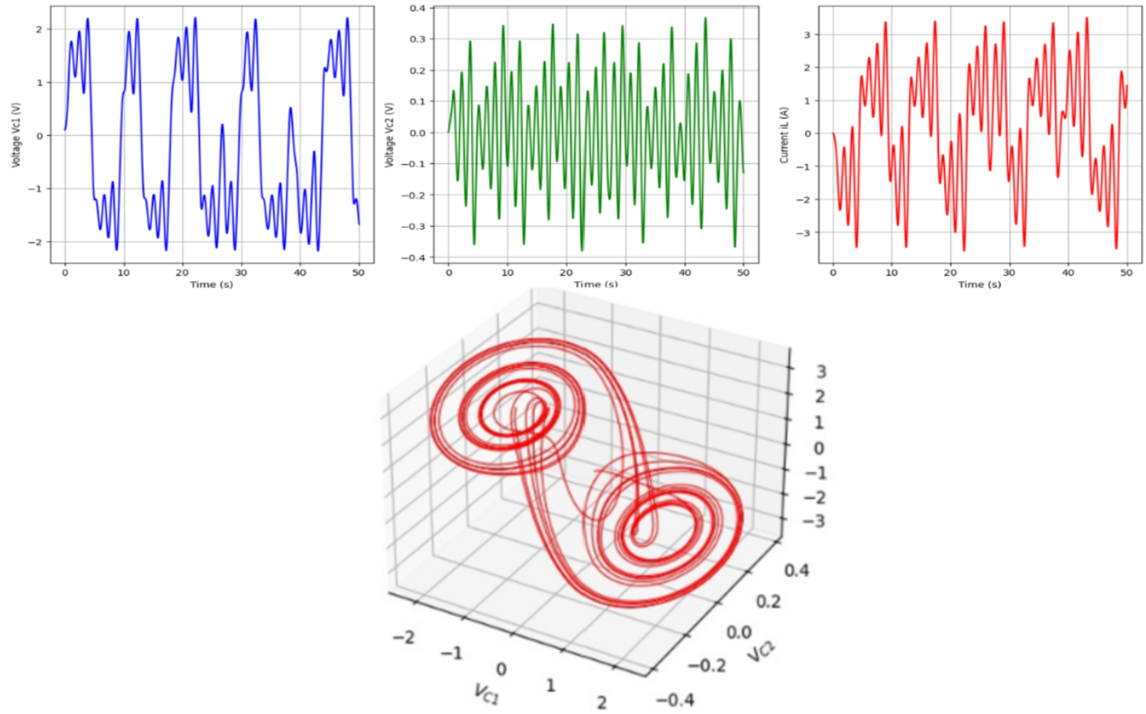


Figure 5: Simulation results of the chaotic circuit showing the time-domain waveforms (top) and the corresponding double-scroll chaotic attractor in 3D phase space (bottom).

5 Discussion and conclusion

The results demonstrate a clear progression from order to chaos as nonlinearity is introduced into the system. The linear RLC circuit, governed by linear differential equations, exhibits only damped or simple harmonic oscillations, predictable behavior where small changes in initial conditions produce small changes in outcomes. This is consistent with classical circuit theory.

The introduction of Chua's diode, a nonlinear element with piecewise-linear V–I characteristics, fundamentally alters the system dynamics. The nonlinearity creates conditions for the emergence of the double-scroll attractor, a bounded region in phase space where the trajectory neither escapes nor repeats. This is the hallmark of deterministic chaos. This is the hallmark of deterministic chaos.

The three state variables V_{C1}, V_{C2}, and i_L are coupled through the circuit equations, explaining why their peaks align despite measuring different physical quantities. This coupling is essential for the coherence of the attractor structure.

6 Future works

This report covers the foundational theory and simulation of the Chua circuit. For this stage, we utilized specific parameter values provided by our supervisor. Future work will involve measuring the physical components of the actual circuit in the lab to get their exact values. These real-world measurements will then be used to create a more detailed simulation.

7 Acknowledgments

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